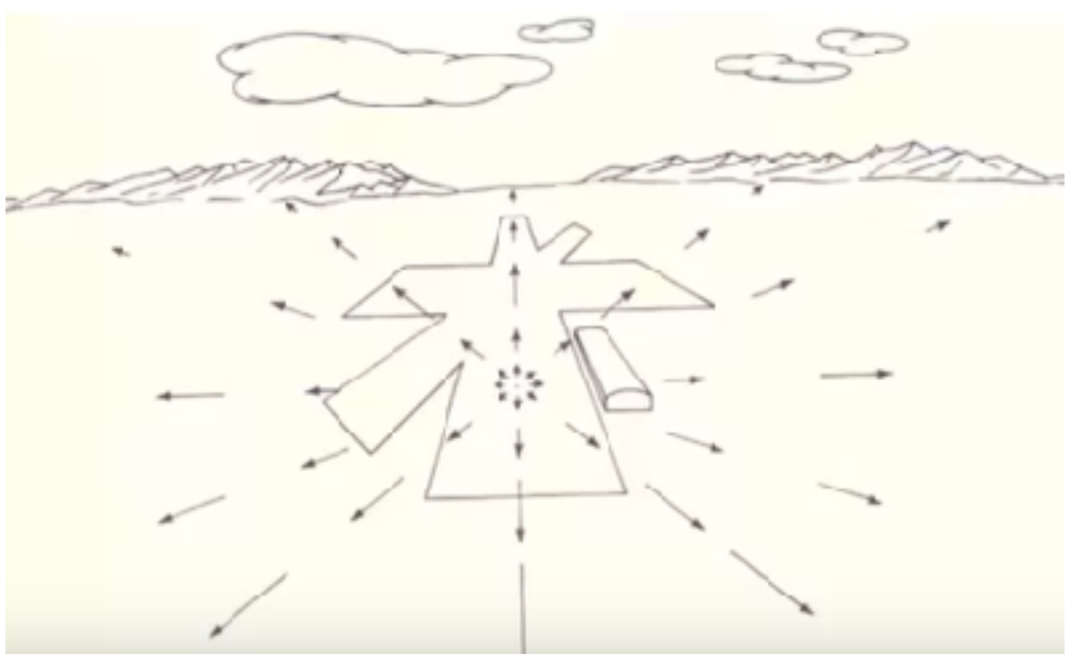
Estimating optical flow

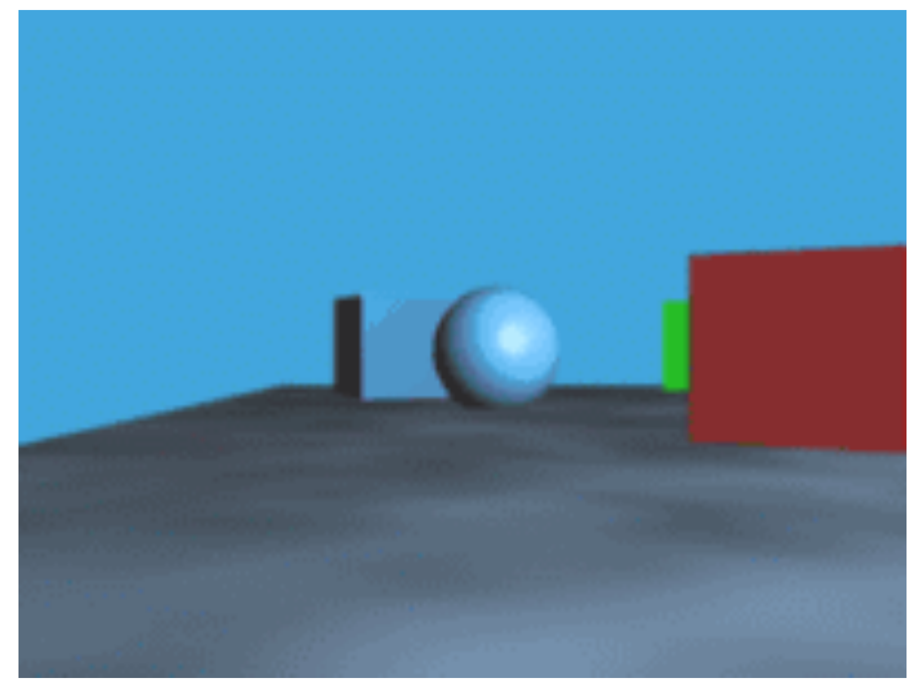
Outline

- Bightness constancy
- Aperture problem
- Small-motion assumption
- Motion segmentation

Last lecture:
biological importance of optical flow

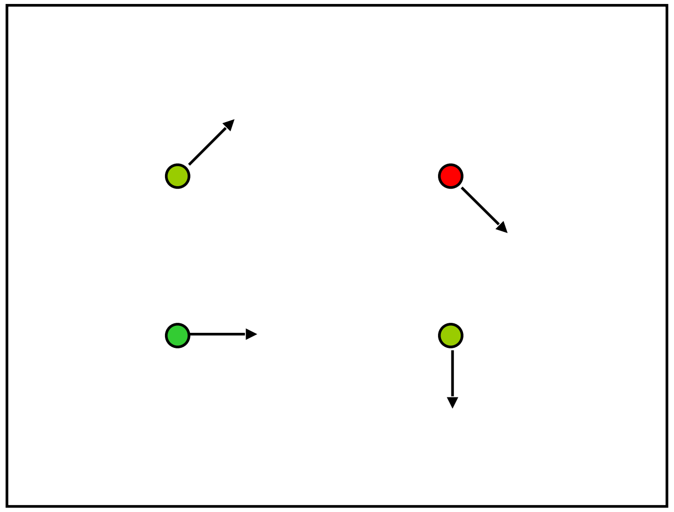


Time-to-contact

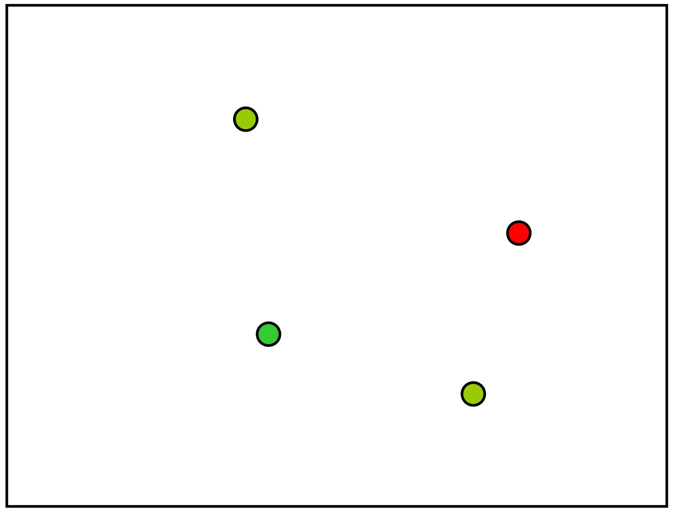


Parallax reveals depth

Problem Definition: Optical Flow



$H(x, y)$



$I(x, y)$

- How to estimate pixel motion from image H to image I?
  - Find pixel correspondences
    - Given a pixel in H, look for nearby pixels of the same color in I
- Key assumption
  - **color constancy**: a point in H looks “the same” in image I
    - For grayscale images, this is **brightness constancy**

Brightness constancy

$$I(x + \Delta x, y + \Delta y, t + \Delta t) - I(x, y, t) = 0$$

$$\frac{\partial I}{\partial x}\Delta x + \frac{\partial I}{\partial y}\Delta y + \frac{\partial I}{\partial t}\Delta t \approx 0$$

$$\frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \frac{\partial I}{\partial t} = 0 \quad \text{where} \quad u = \frac{\Delta x}{\Delta t}, v = \frac{\Delta y}{\Delta t}$$

$$\nabla I \cdot \begin{bmatrix} u \\ v \end{bmatrix} + \frac{\partial I}{\partial t} = 0$$

Brightness constancy equation gives us:

1) a constraint on flow vector (u,v)
2) a linear approximation of pixel error

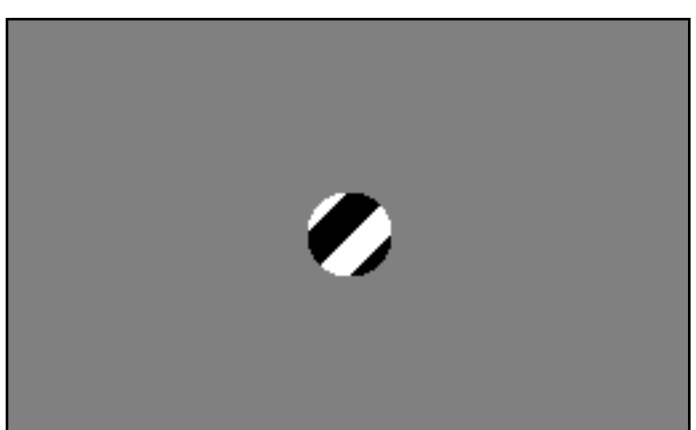
Aperature problem

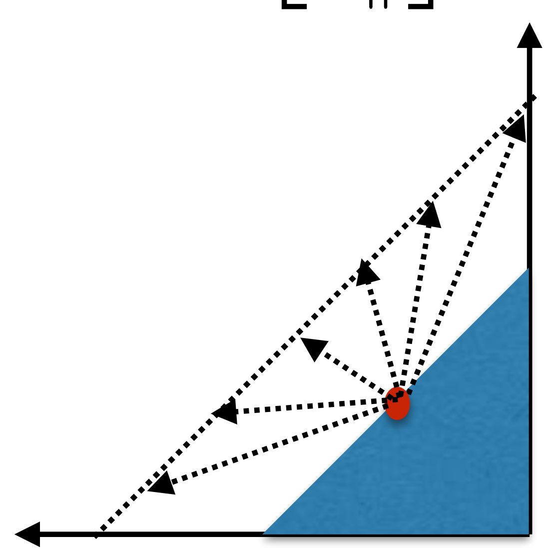
We can only determine flow in direction parallel to gradient

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} u_\perp \\ v_\perp \end{bmatrix} + \begin{bmatrix} u_\| \\ v_\| \end{bmatrix}$$

$$\nabla I \cdot \begin{bmatrix} u \\ v \end{bmatrix} = \nabla I \cdot \begin{bmatrix} u_\perp \\ v_\perp \end{bmatrix} + \nabla I \cdot \begin{bmatrix} u_\| \\ v_\| \end{bmatrix}$$

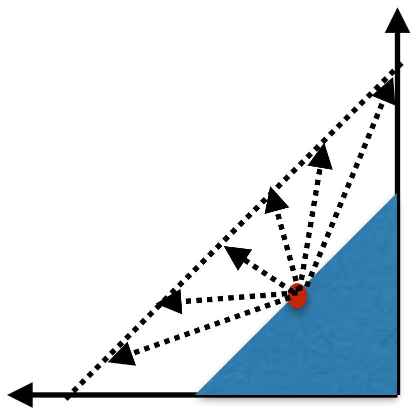
$$= \nabla I \cdot \begin{bmatrix} u_\| \\ v_\| \end{bmatrix}$$





Challenges

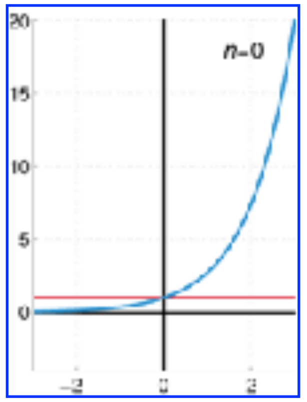
- Aperture problem



Soln to brightness constancy equation may not be unique

$$I(x, y, t) = I(x + \Delta x, y + \Delta y, t + \Delta t)$$

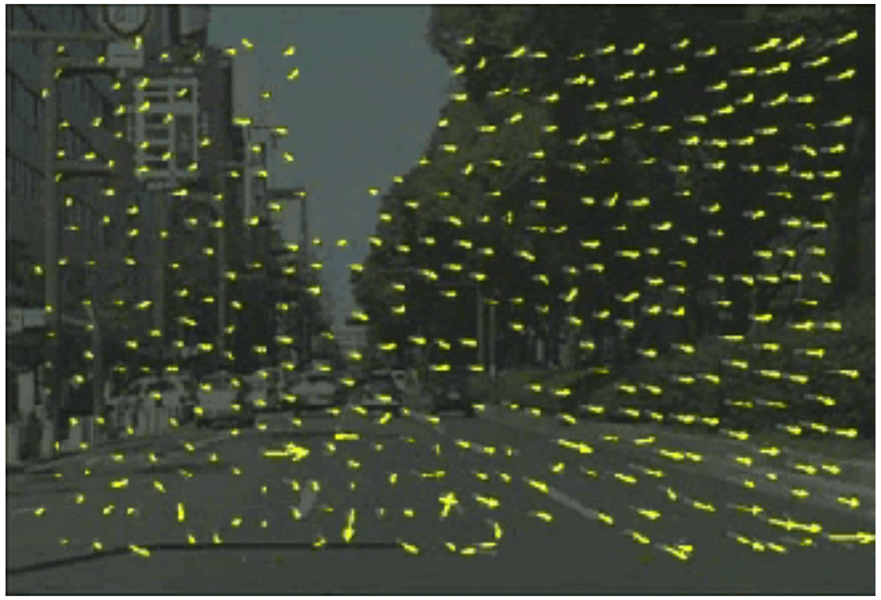
- Small motion assumption



First-order taylor approximation does not hold for large motions

$$\nabla I \cdot \begin{bmatrix} u \\ v \end{bmatrix} + \frac{\partial I}{\partial t} = 0$$

Soln for aperture problem

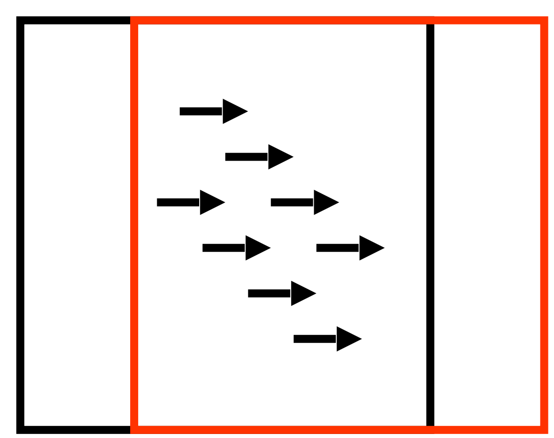


1. Don't try to estimate flow at unreliable points (sparse flow; similar to feature point alignment!)

2. Assume neighboring flow vectors are similar (enforce *spatial smoothness* in dense flow feild)

Simple approach: assume flow is constant over a neighborhood

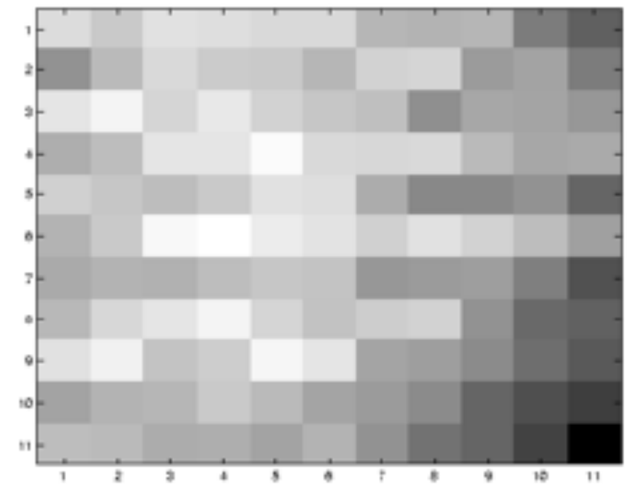
$$\min_{u,v} \sum_{x,y \in W} \Big(I_2(x+u, y+v) - I_1(x,y) \Big)^2$$

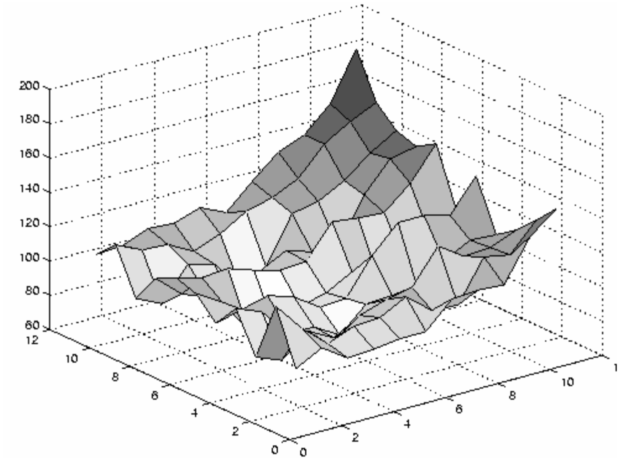


$$u(x,y) = u$$

$$v(x,y) = v$$

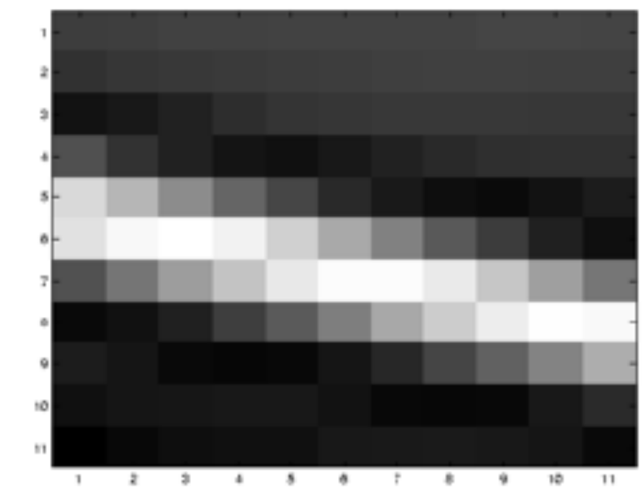
Low Texture Region - Bad

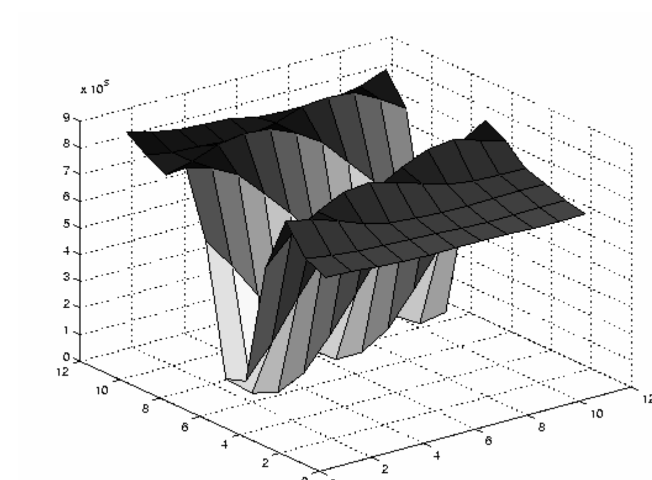




SSD surface

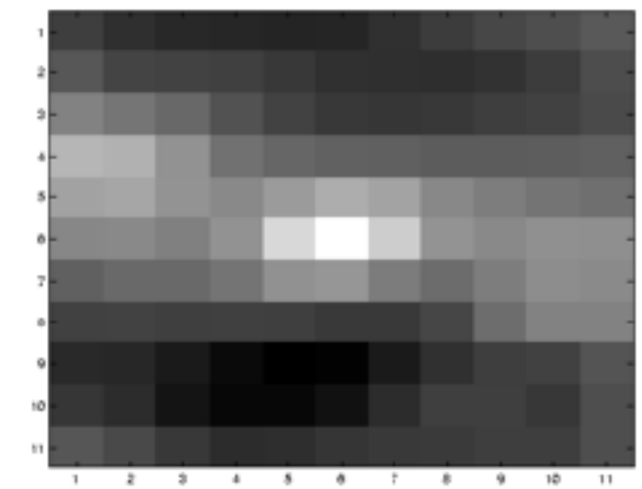
Edges – so,so (aperture problem)

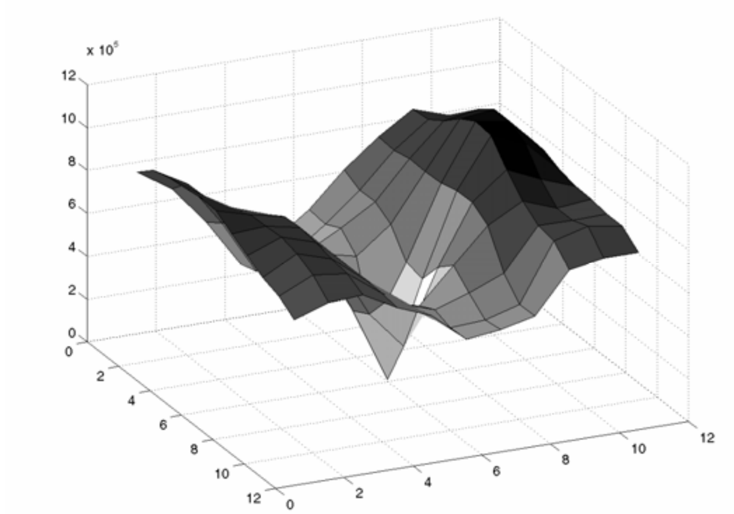




SSD surface

High Textured Region - Good





SSD surface

Sparse flow estimation (feature tracking)

1. User Harris corner score to find trackable patches

$$I_2(x+u, y+v) - I_1(x,y) \approx \nabla I(x,y) \begin{bmatrix} u \\ v \end{bmatrix} + I_t(x,y)$$

2. Appy Lucas Kanade on those patches

Good Features to Track

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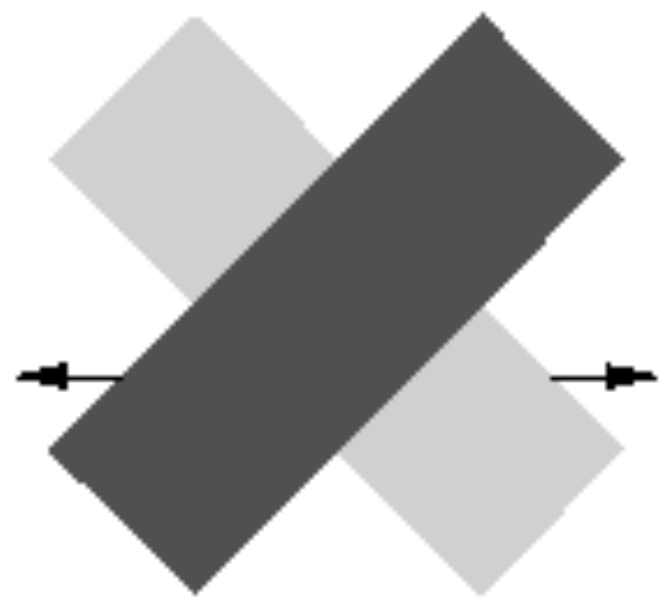
Local motion estimation is hard

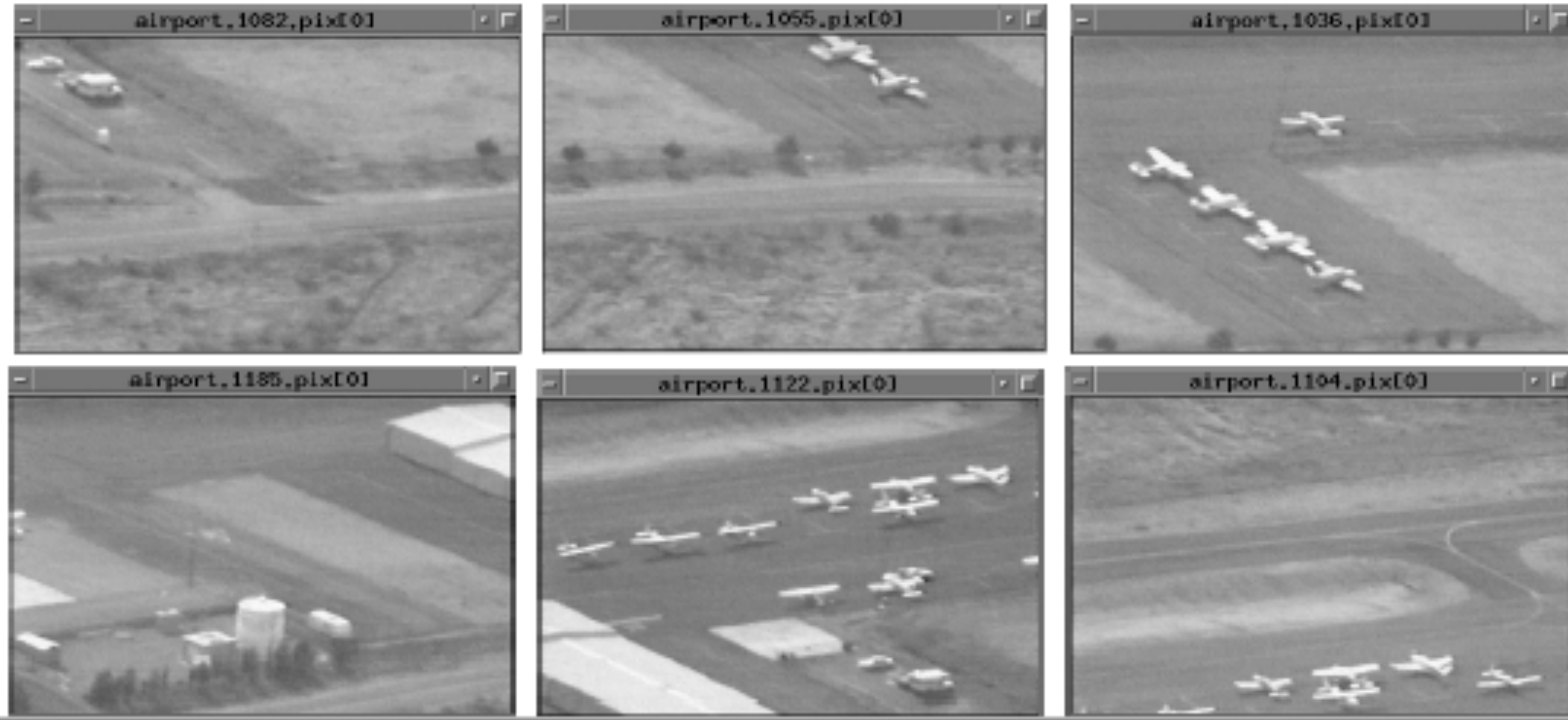


Where does false “t-junctions” appear to move?

We’d like to integrate local signals globally

Dense flow (I)

(a)



$$E(\mathbf{p}) = \sum_{\mathbf{x}} [I(\mathbf{W}(\mathbf{x};\mathbf{p})) - T(\mathbf{x})]^2$$

Apply Lucas Kanade on successive frames of a video sequence

Generalize translation to other 2D warps (affine, homographies,…)

Applications: mosaicing

Homography warp works for some cases (rotations, planar scenes).
We’ll discuss a solution for others in a bit…

Dense flow (II)

Solve for global flow feild

$$\min_{\substack{u(x,y)\\v(x,y)}} \sum_{x,y} [I_2(x+u(x,y), y+v(x,y)) - I_1(x,y)]^2$$

Aside: continuous case

$$\min_{u,v} \int\int \big(I_2(x+u, y+v) - I_1(x,y)\big)^2 dxdy$$

Formal math is known as calculus of variations (we're minimizing over the *space of functions*)

https://en.wikipedia.org/wiki/Calculus_of_variations

Dense *variational* flow

If we assume small motions….

$$I_2(x+u, y+v) - I_1(x,y) \approx \nabla I \cdot \begin{bmatrix} u \\ v \end{bmatrix} + I_t$$

$$\min_{u,v} \int \int (\nabla I \cdot \begin{bmatrix} u \\ v \end{bmatrix} + I_t)^2 dxdy$$

above is "shorthand" for…

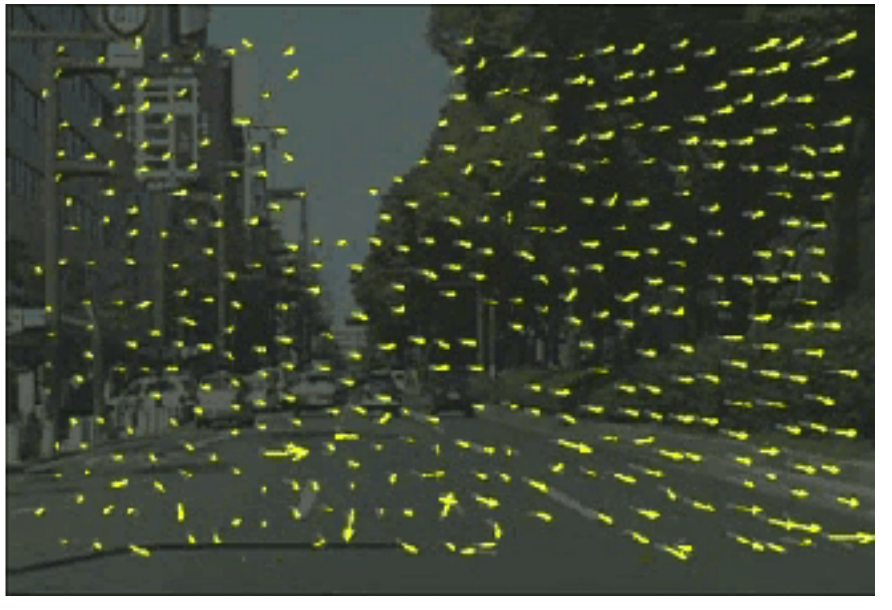
$$\min_{\substack{u(x,y) \\ v(x,y)}} \sum_{x,y} \left[\nabla I(x,y) \cdot \begin{bmatrix} u(x,y) \\ v(x,y) \end{bmatrix} + I_t(x,y) \right]^2$$

Spatial regularization

Penalize differences in nearby flow vectors

$$\min_{u,v} E_{intensity} + E_{smooth}$$

$$E_{intensity}(u,v) = \int\int (\nabla I \cdot \begin{bmatrix} u \\ v \end{bmatrix} + I_t)^2 dxdy \qquad E_{smooth}(u,v) = \int\int ||\nabla u||^2 + ||\nabla v||^2 dxdy$$



1. Unknowns (u,v) appear quadratically in above expression => discretize above and solve for them with a giant linear system

$$\min_{\substack{u(x,y)\\ v(x,y)}} \sum_{x,y} \Big(I_x(x,y)u(x,y) + I_y(x,y)v(x,y) + I_t(x,y)\Big)^2 + (u(x+1,y) - u(x,y))^2 + \dots$$

2. Challenge: outliers will overwhelm squared error term

Challenges with regularization

https://en.wikipedia.org/wiki/Horn-Schunck_method

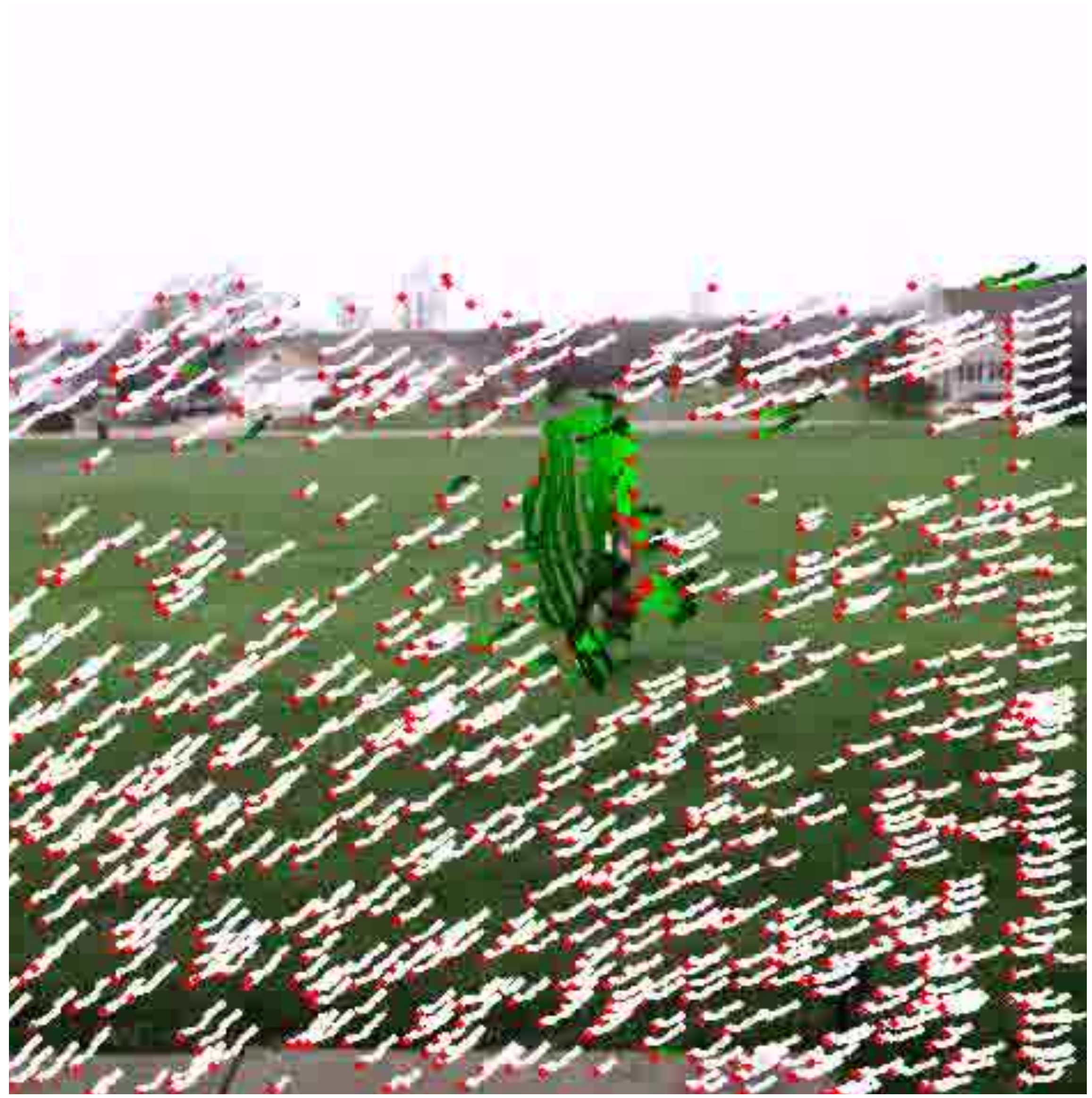
$$\min_{u,v} E_{intensity} + E_{smooth}$$

estimated flow ground-truth flow

$$E_{intensity}(u,v) = \int\int (\nabla I \cdot \begin{bmatrix} u \\ v \end{bmatrix} + I_t)^2 dxdy$$

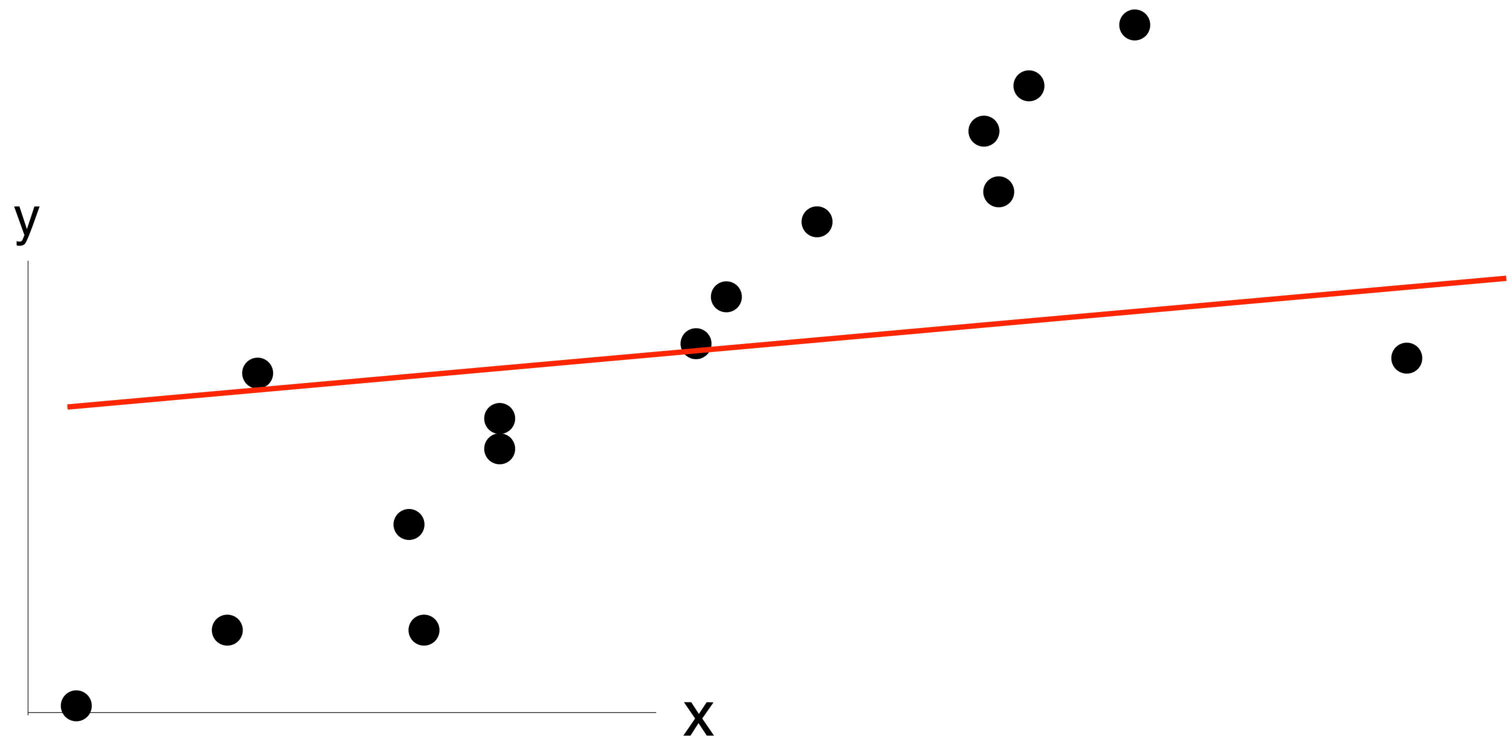
$$E_{smooth}(u,v) = \int\int ||\nabla u||^2 + ||\nabla v||^2 dxdy$$

Can we use ransac to fit flow?

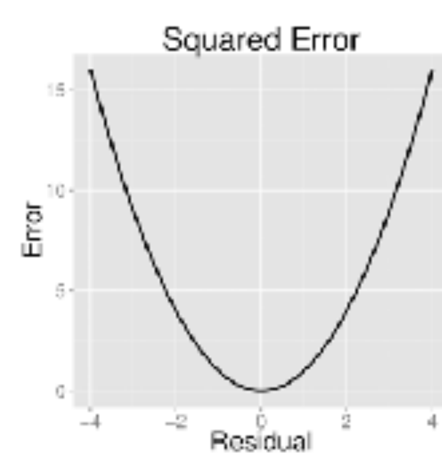


Yes, but historically optical flow methods take a more continuous optimization (variational) perspective

Recall RANSAC

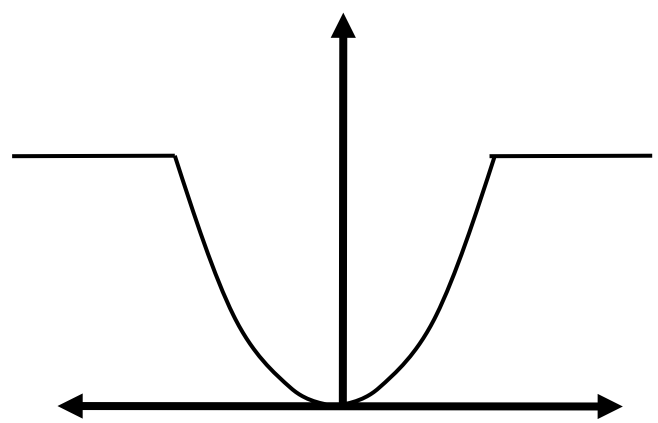


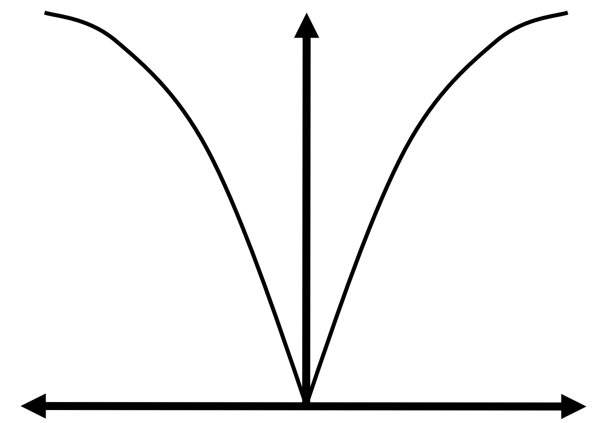
Underlying problem: squared error penalizes outliers too much



Deeper reason: Gaussian statistics ($e^{-||x-u||^2}$) are too simplistic for real world

RANSAC as robust model-fitting





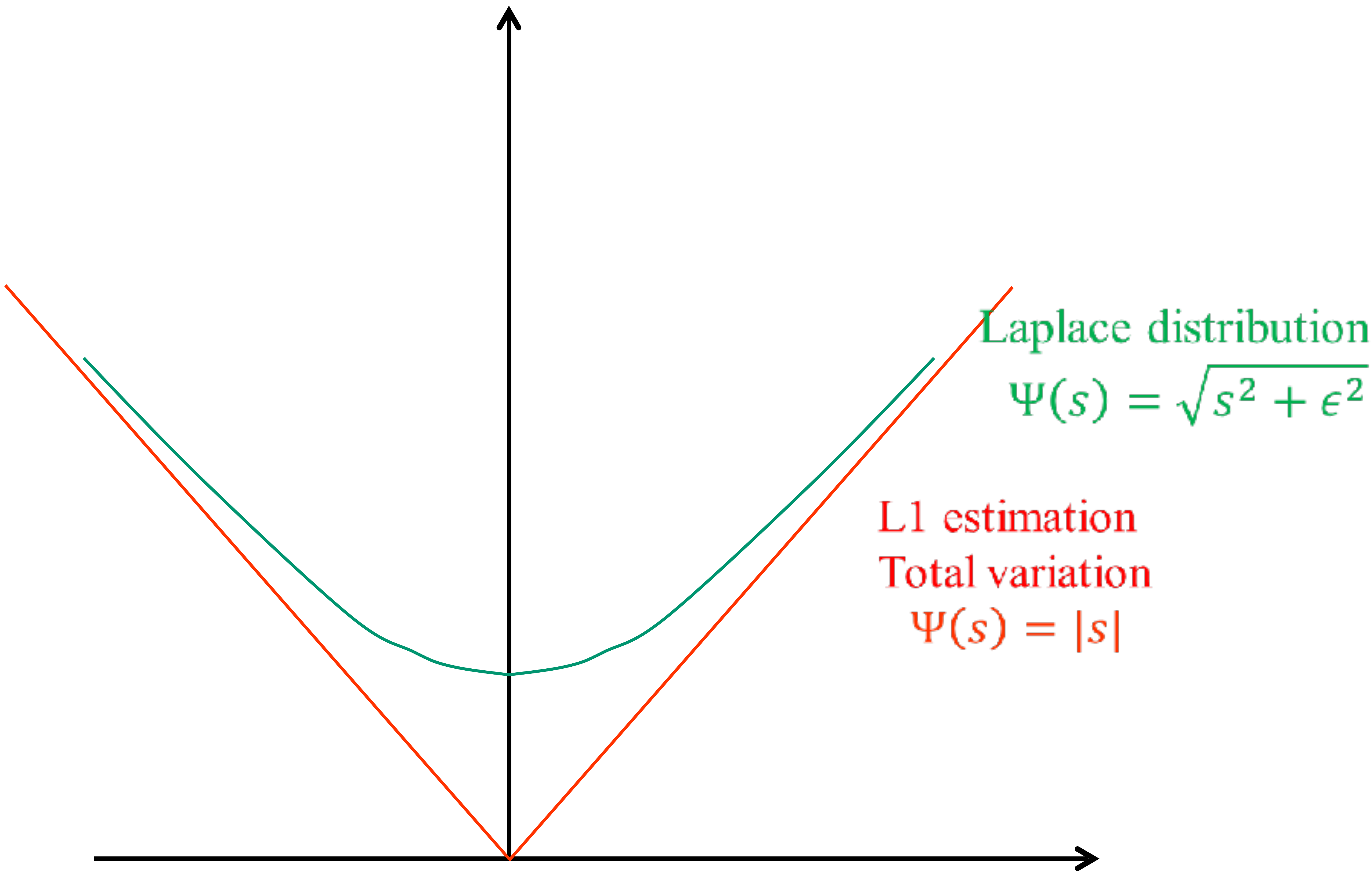
$$\rho_{\alpha,\lambda}(x) = \begin{cases} \lambda x^2 & \text{if } |x| < \frac{\sqrt{\alpha}}{\sqrt{\lambda}} \\ \alpha & \text{otherwise} \end{cases} \qquad \rho_\sigma(x) = log\left(1 + \frac{1}{2}(\frac{x}{\sigma})^2\right)$$

Optimization over model parameters (u,v) not convex

It turns out, we can generalize RANSAC to an algorithm for *maximum-likelihood* model fitting robust error models (MLESAC, Torr & Zisserman 00)

Instead of scoring a candidate model with # of inliers, score it under robust error function

“Intermediate” approach: robust statistics that are convex



Energy function(u,v) is still globally optimizable with local search

Robust variational optical flow

$$\min_{u,v} \int\int \rho(I_2(x+u, y+v) - I_1(x,y)) + \rho(||\nabla u||) + \rho(||\nabla v||) dxdy$$

where rho = robust error function (instead of quadratic error)

first image quadratic flow lorentzian flow detected outliers

Typical approach: initialize solution with quadratic flow and locally optimize

Reference

- Black, M. J. and Anandan, P., A framework for the robust estimation of optical flow, *Fourth International Conf. on Computer Vision* (ICCV), 1993, pp. 231-236 http://www.cs.washington.edu/education/courses/576/03sp/readings/black93.pdf

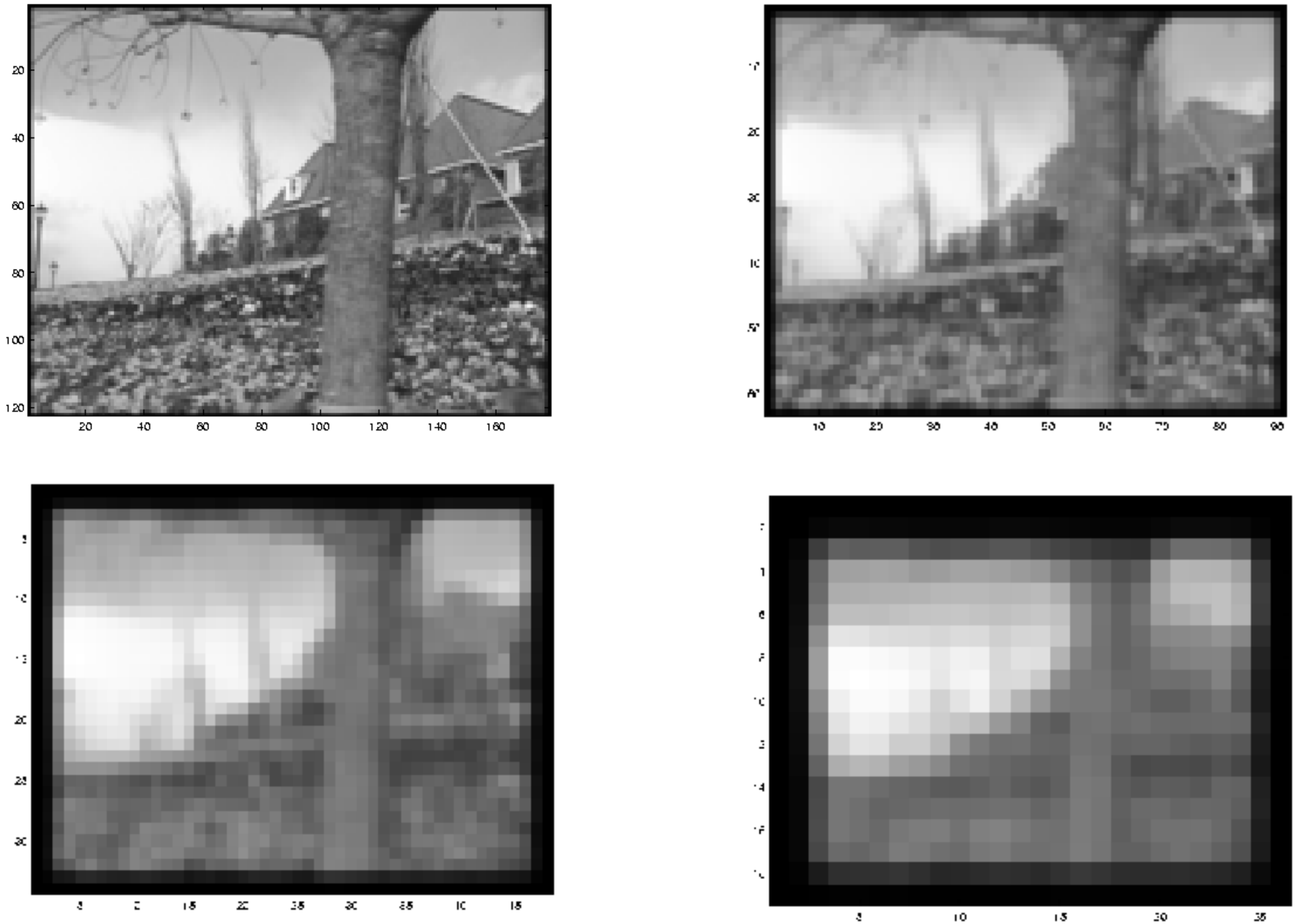
Outline

- Bightness constancy
- Aperture problem (sparse flow, spatial regularization)
- Small-motion assumption
- Motion segmentation

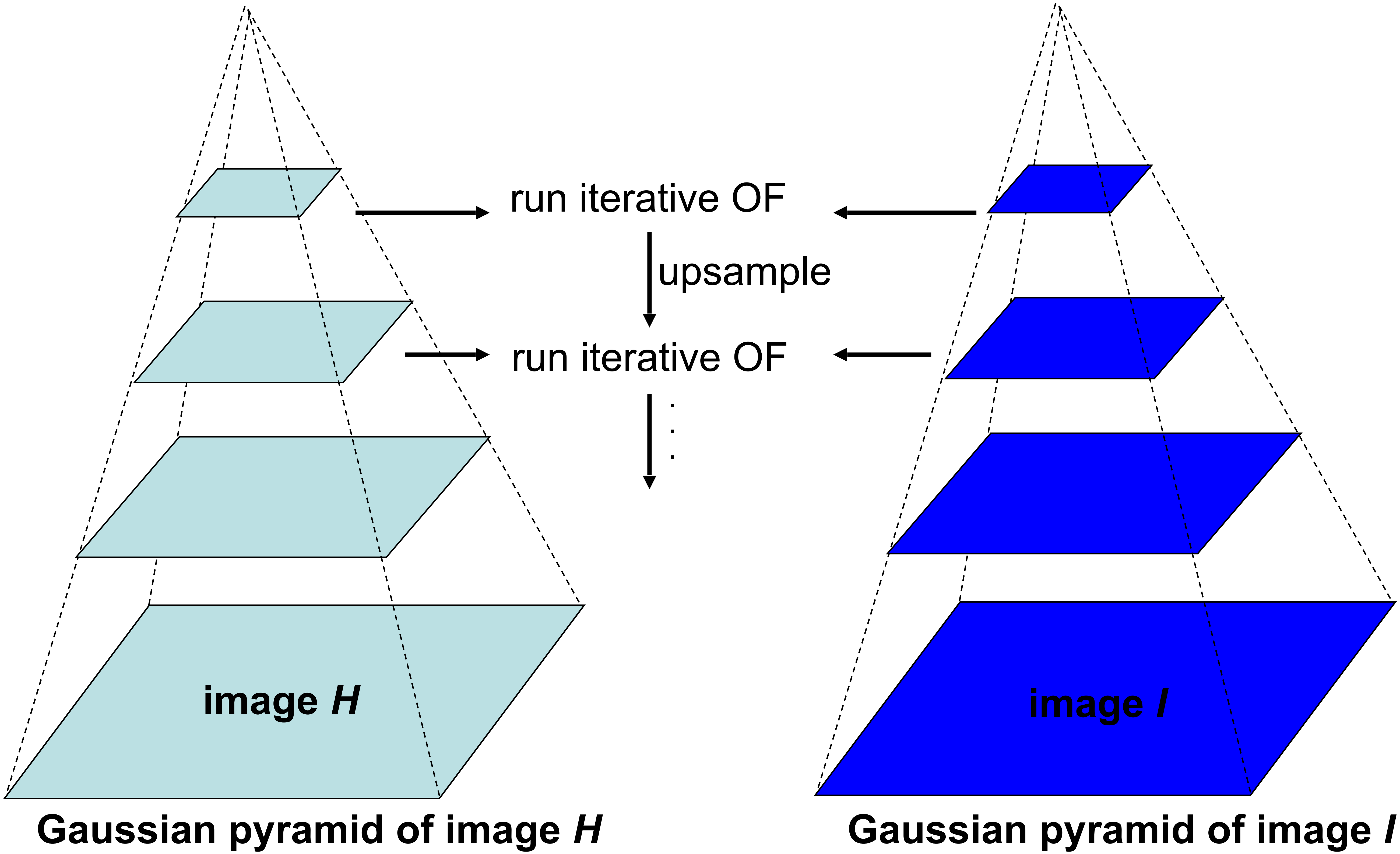
Revisting the small motion assumption

Is the motion small enough to make Taylor-series linearization valid?

One soln: reduce the resolution!



Soln 1: Coarse-to-fine Optical Flow

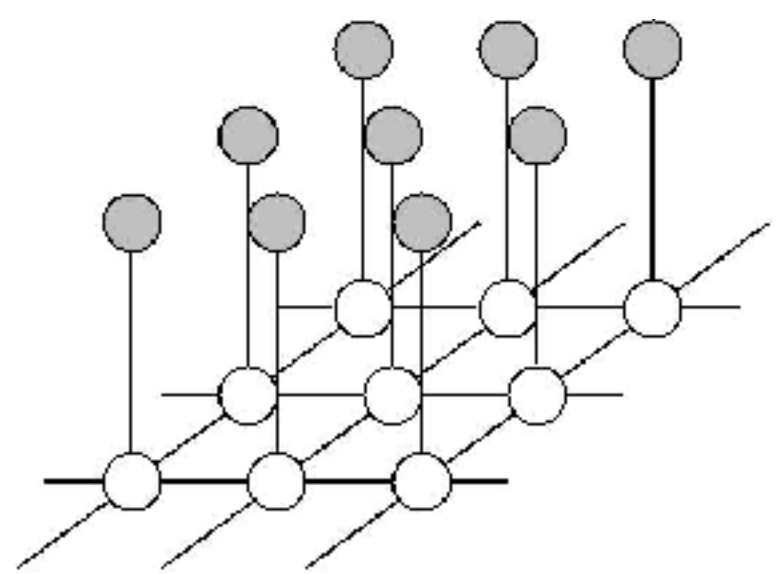


Soln 2: discrete optical flow estimation

$$u_i \in \{-5 \ldots 5\}$$
$$v_i \in \{-5 \ldots 5\}$$
$$z_i = (u_i, v_i)$$

$$\phi_i(z_i) = \rho(||I_2(x_i + u_i, y_i + v_i) - I(x_i, y_i)||)$$
$$\psi_{ij}(z_i, z_j) = \rho(u_i - u_j, v_i - v_j)$$

$$E(z) = \sum_{i \in V} \phi_i(z_i) + \sum_{ij \in E} \psi_{ij}(z_i, z_j)$$



Discrete Markov Random Feild (MRF) with pixel-grid graph G=(V,E)

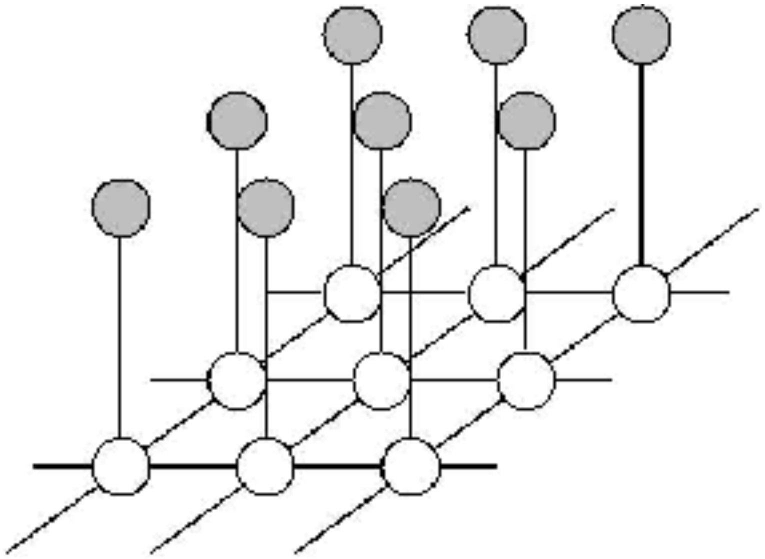
A Database and Evaluation Methodology for Optical Flow

Simon Baker · Daniel Scharstein · J.P. Lewis ·
Stefan Roth · Michael J. Black · Richard Szeliski

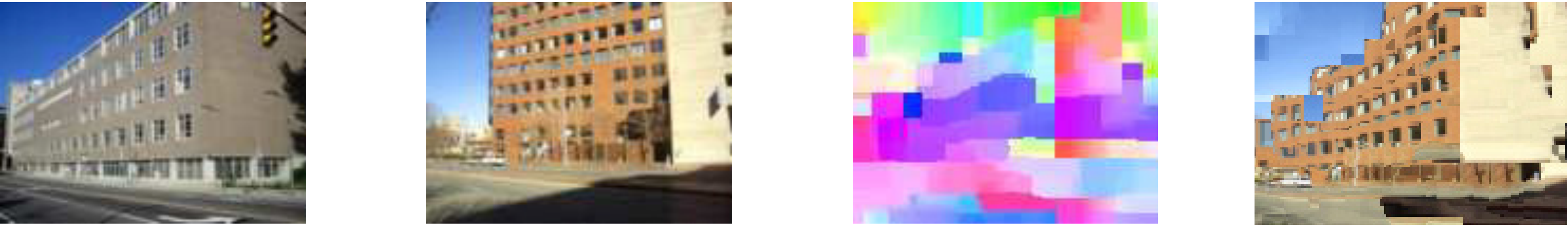
Example: SIFTFlow

Measure local appearances of patches using SIFT descriptors

Turns out that this can be used to align images of different scenes!



Liu et al, PAMI 2011



Allows us to do nearest-neighbor label transfer for scene analysis

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[Some remaining challenges]

Remaining challenges: long-term optical flow

Combine long-term sparse feature tracking with variational flow regularization

(http://rvsn.csail.mit.edu/pv/)

Note the difficulty in getting regularization "right"!

Remaining challenges: small things that move fast

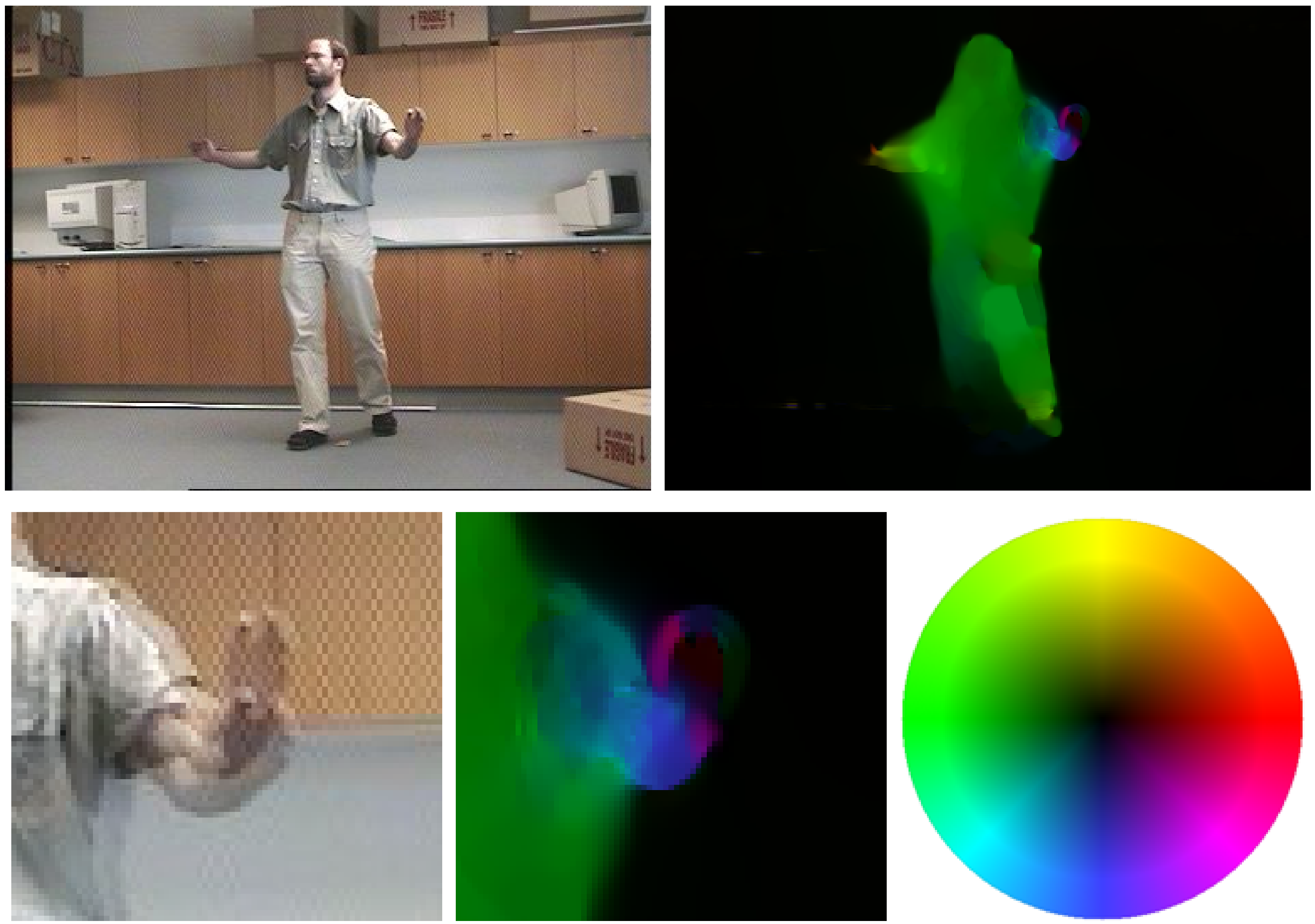

Figure 1. **Top row:** Image of a sequence where the person is stepping forward and moving his hands. The optical flow estimated with the method from [4] is quite accurate for the main body and the legs, but the hands are not accurately captured. **Bottom row,**

Large Displacement Optical Flow*

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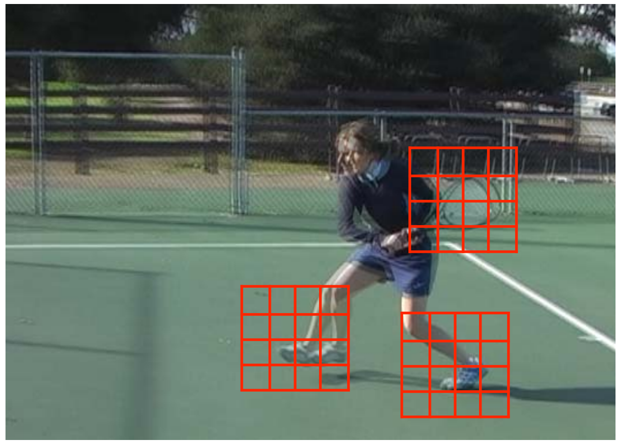
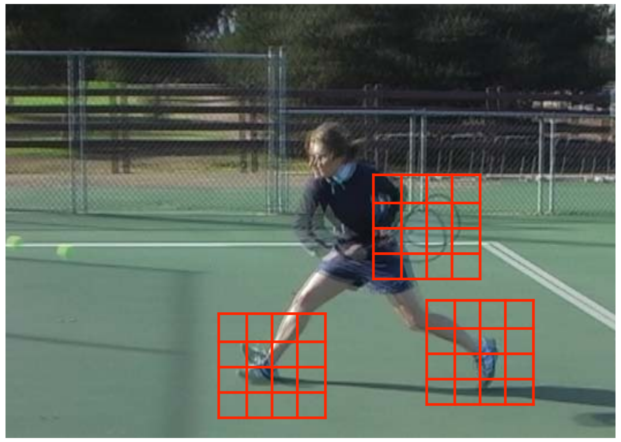
Estimate dense or sparse correspondences across 2 frames with classic descriptor matching



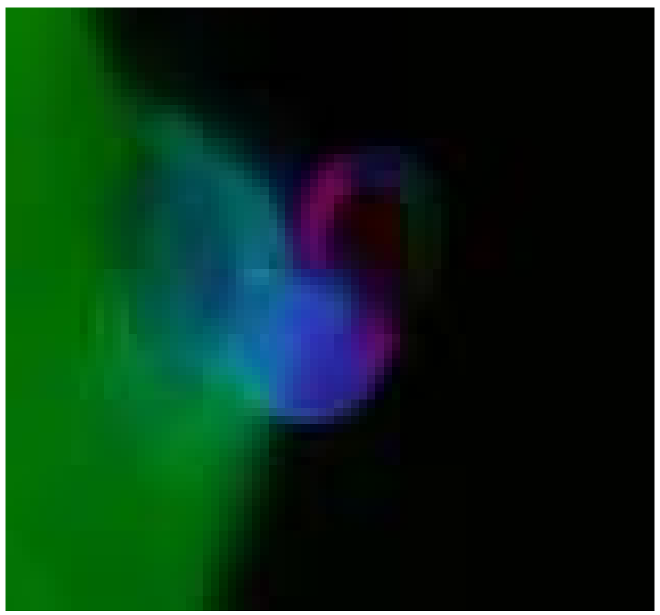


Set of matchable points and estimated offsets: $\{(x_i, y_i, u_i, v_i)\}$

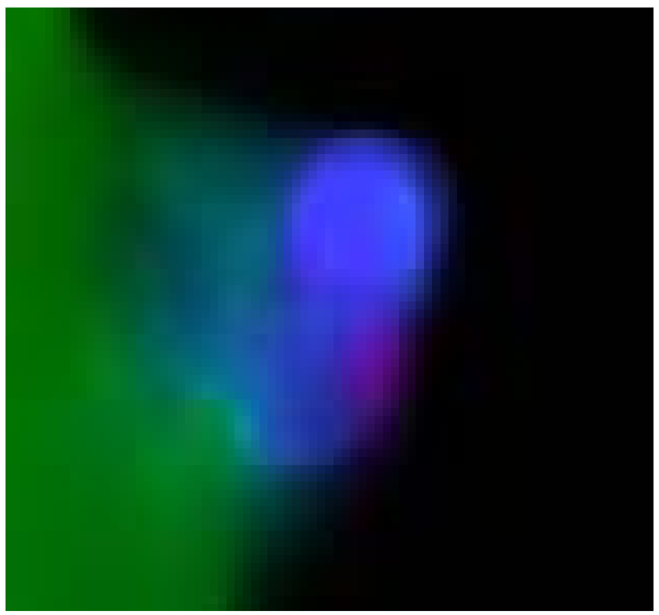
$$E_{match}(u,v) = \sum_i (u(x_i,y_i) - u_i)^2 + (v(x_i,y_i) - v_i)^2$$

$$\min_{u,v} E_{intensity} + E_{smooth} + E_{match}$$

Examples



no E_{match}



with E_{match}

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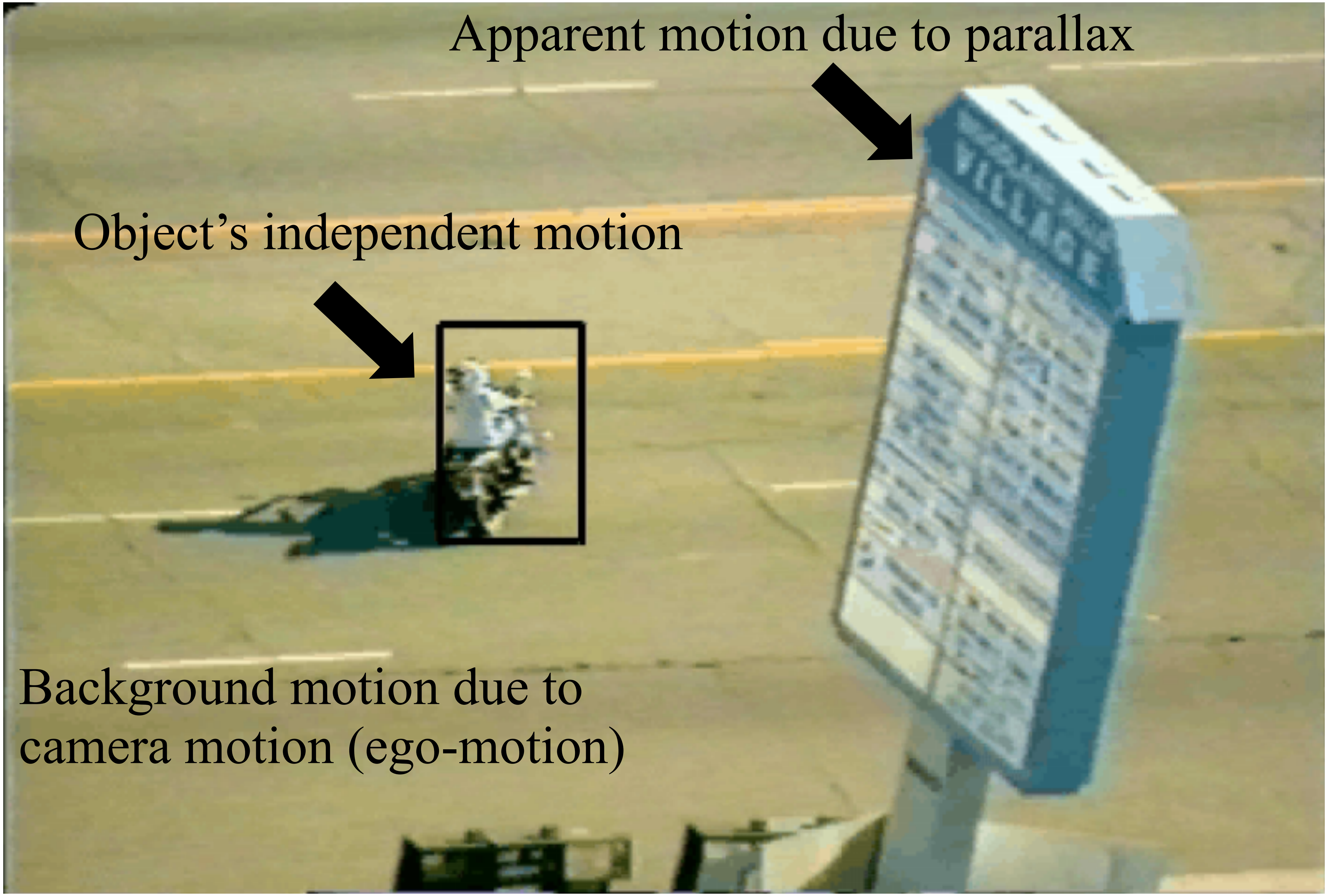
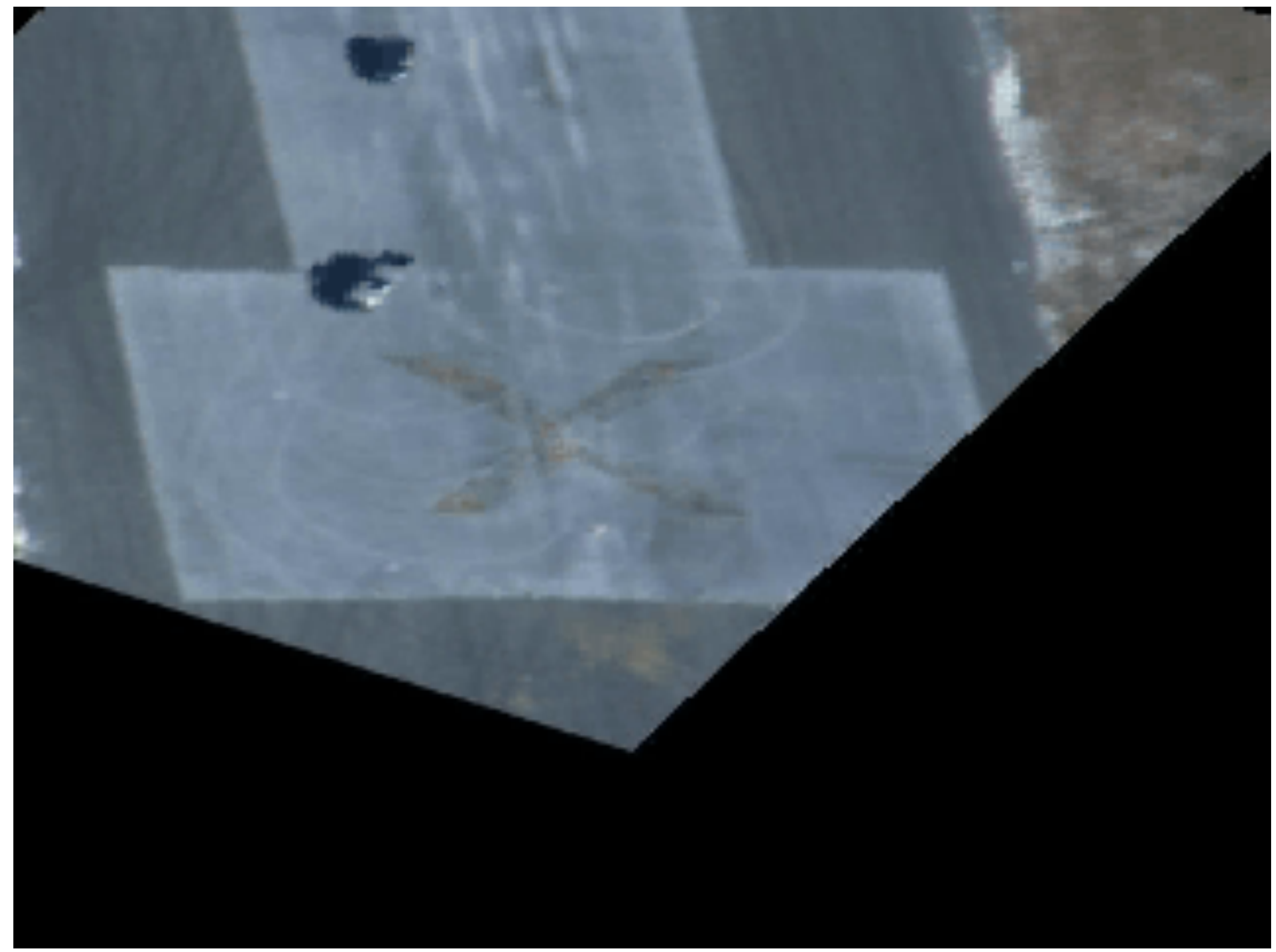

Apparent motion due to parallax
Object's independent motion
Background motion due to camera motion (ego-motion)
VILLAGE

Motion segmentation (I): robustly estimate dominant motion

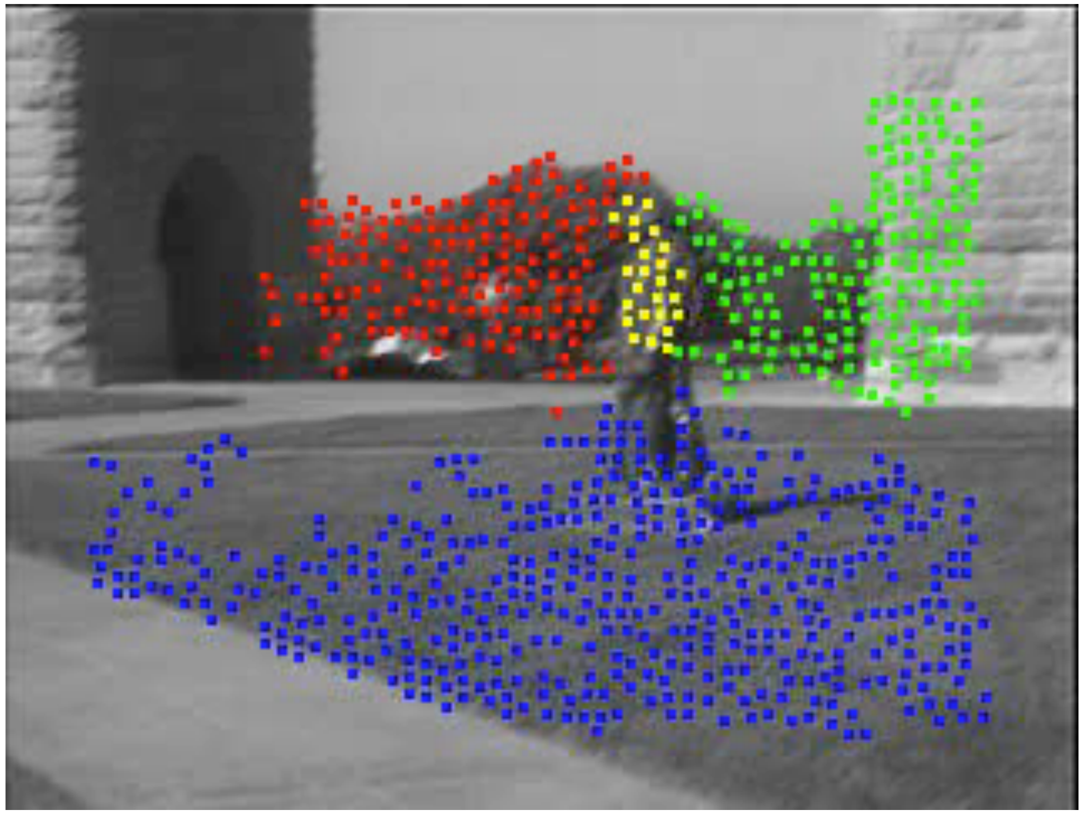
1. Assume parametric warp (typically homography)
2. Treat moving/non-planar objects as outliers in robust error function

$$E(\mathbf{p}) = \sum_{\mathbf{x}} \rho(I(\mathbf{W}(\mathbf{x};\mathbf{p}))) - T(\mathbf{x}))$$

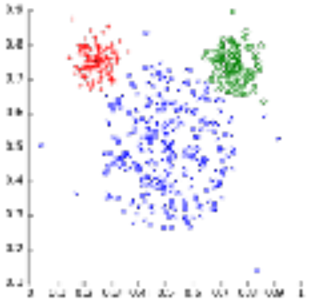
Motion segmentation (II)

Treat as clustering problem



1. Obtain an initial estimate of flow (sparse or dense)
2. Cluster pixels using feature vectors (consisting of flow, RGB, etc.)

Generalize K-means to fit a parametric model (e.g., affine warp) rather than a centroid

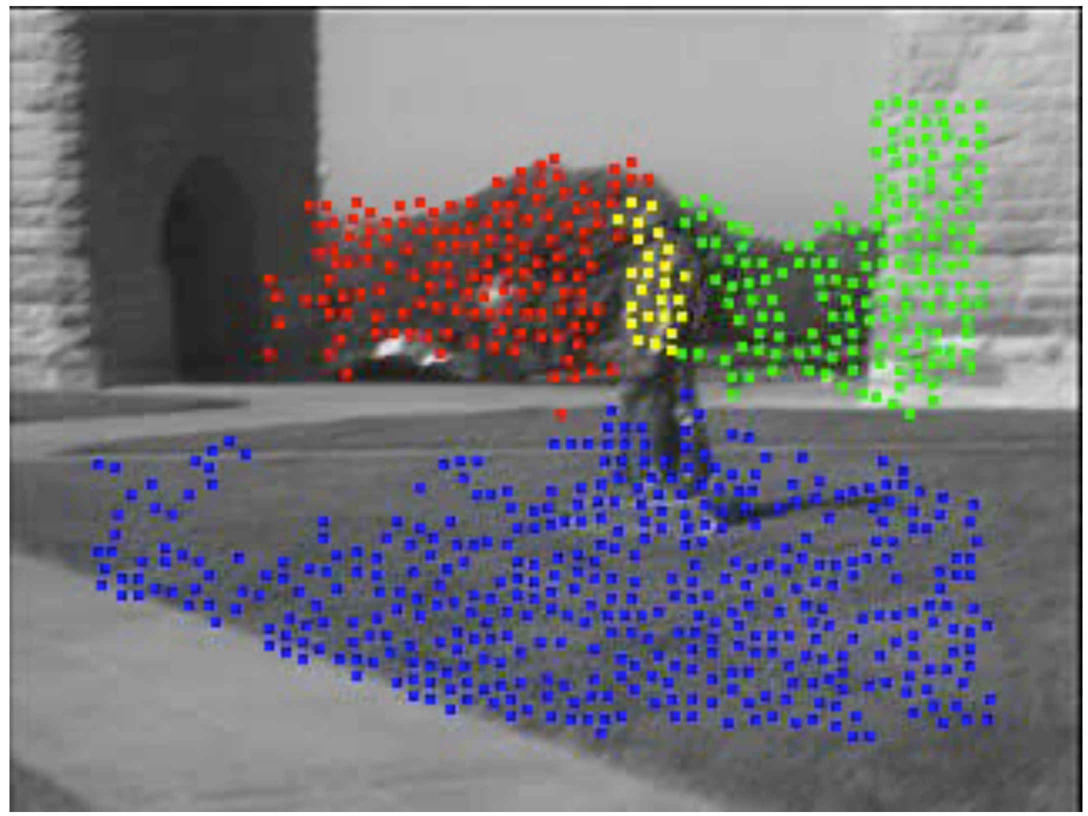


Weiss & Adelson, CVPR 96

Uses “soft” K-means or EM algorithm

Motion segmentation (II)

Treat as clustering problem

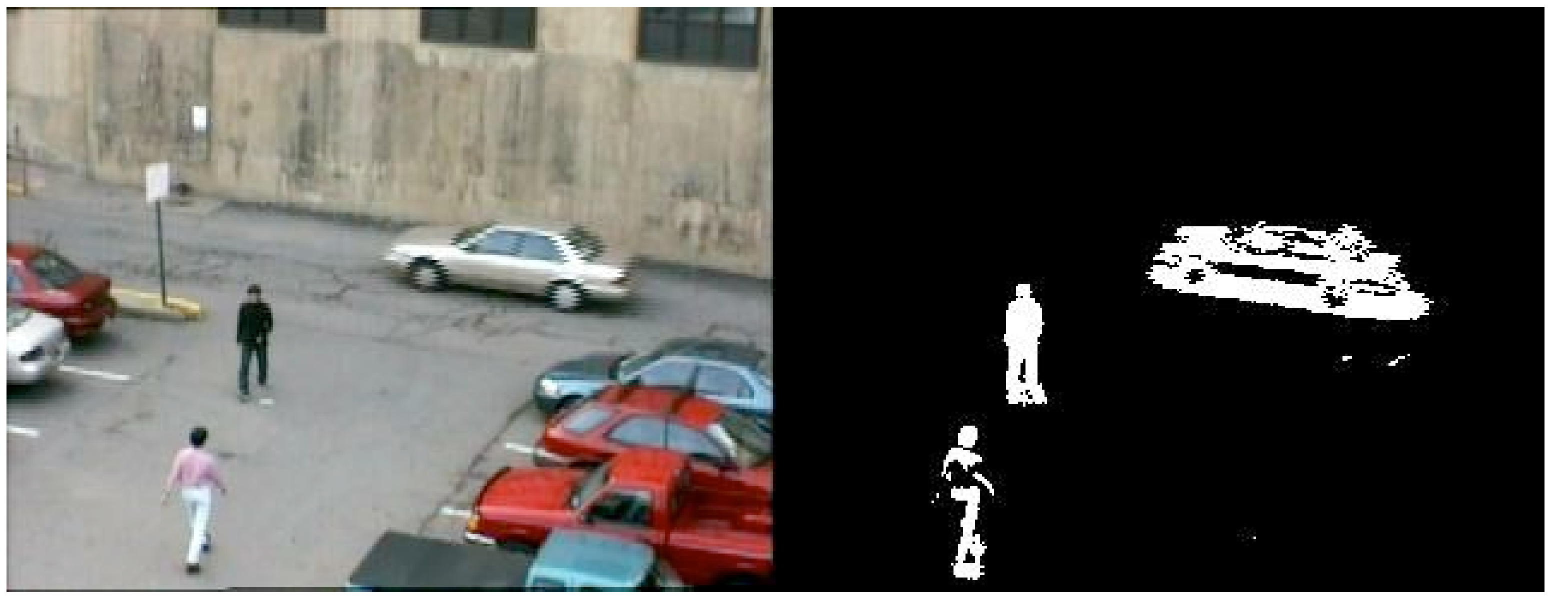


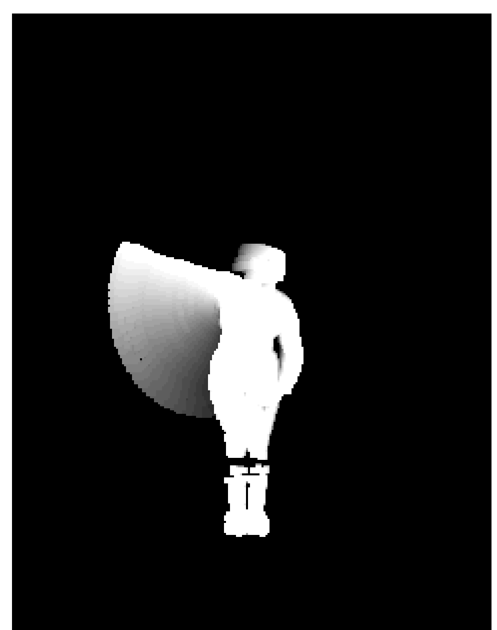
Ideally, estimate flow and warp parameters jointly in one giant variational optimization

(I haven't seen this; looks hard because of joint discrete / continous optimization)

Background subtraction

Once we have background image/mosiac (trivial for a stationary camera), how do we identify foreground?

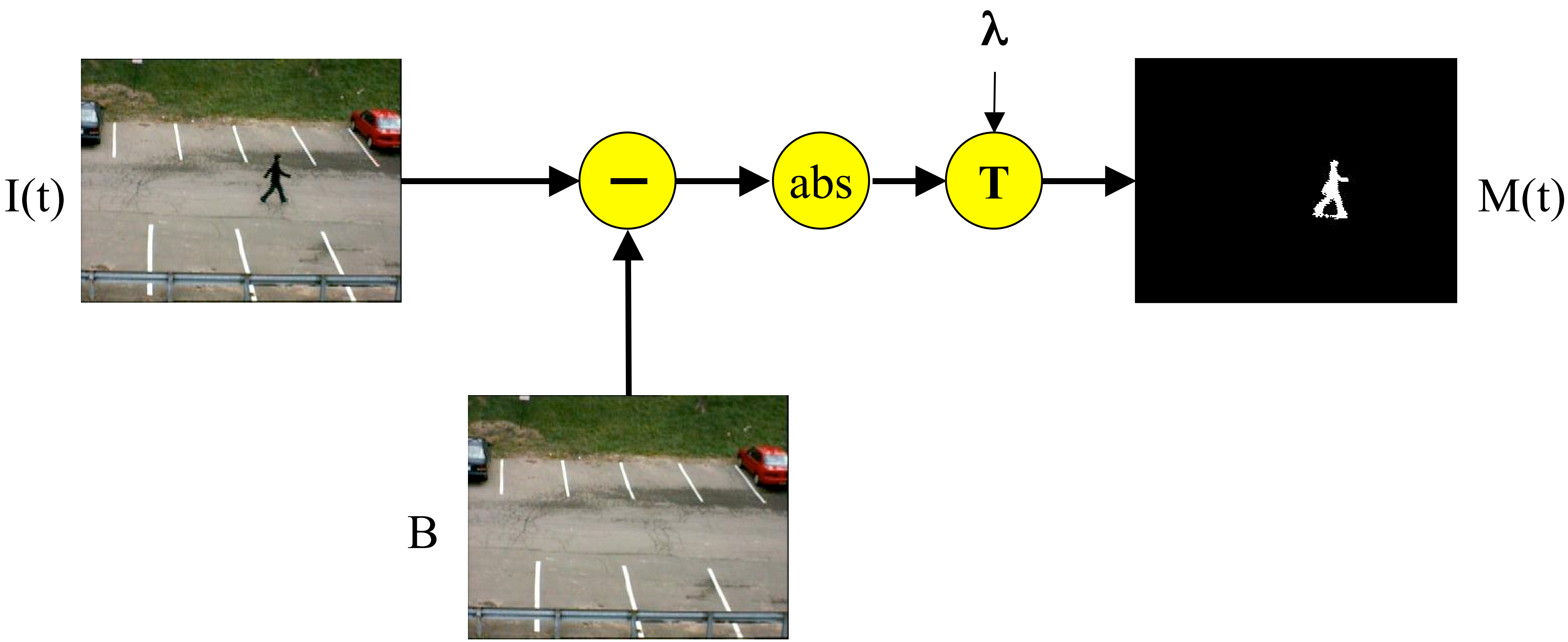




Very commonly-used technique, so we'll spend a few slides on it…

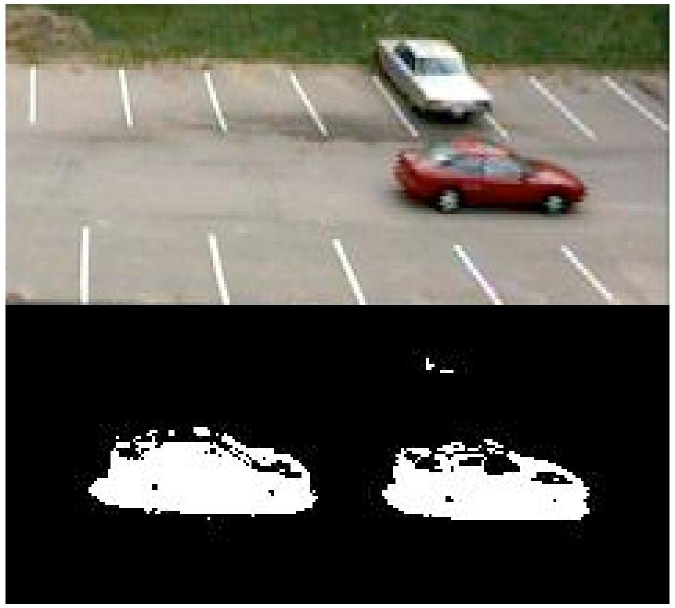
A naive approach

$$M(t) = ||B - I(t)|| > \lambda$$



Difficulties

Overlapping foreground objects are merged together



Formerlly static objects (that now move) result in ghosting

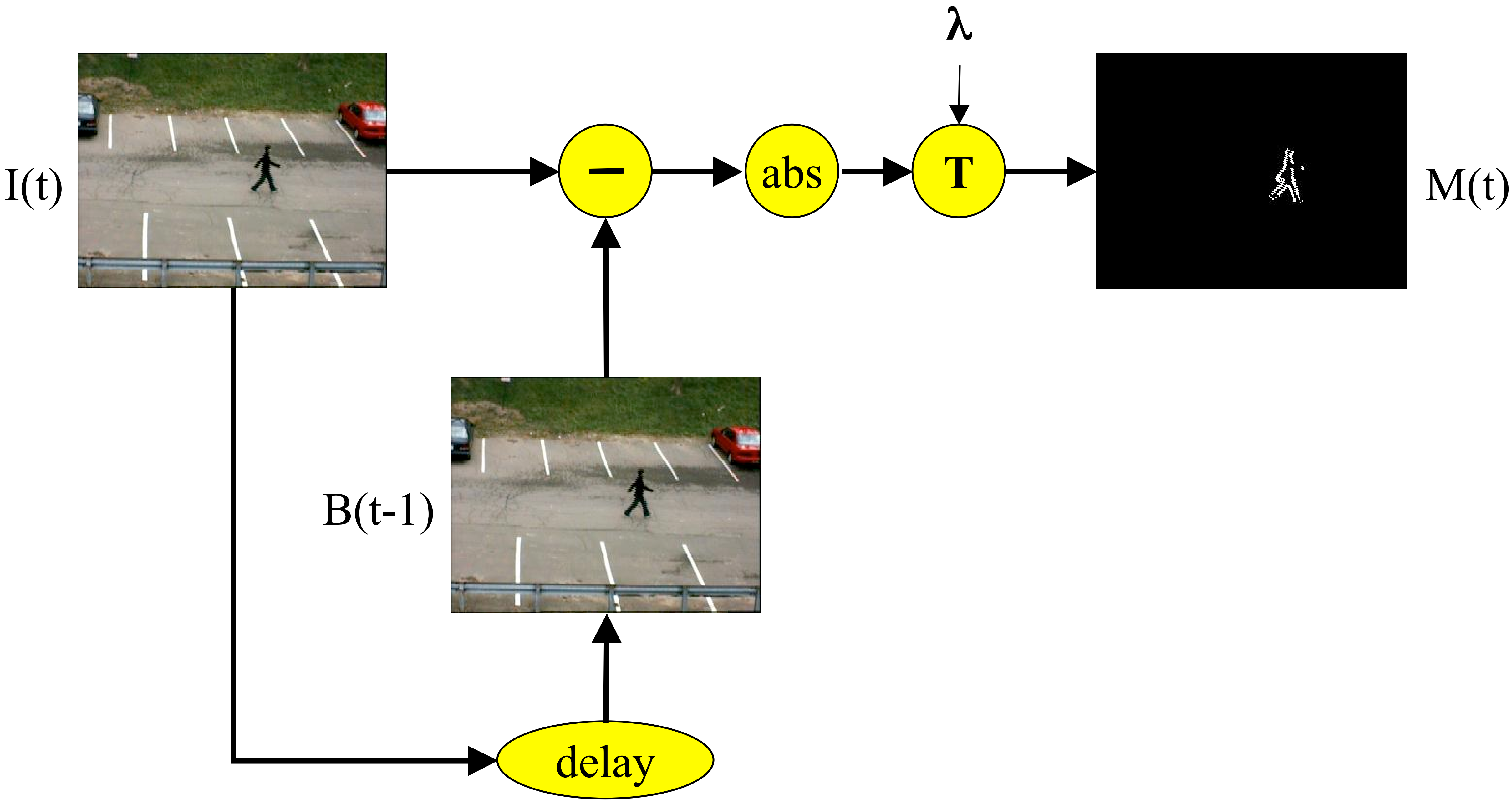
Senstive to small movements in scene (trees) and changes in illumination (sunlight)

Senstive to small movements of camera

Frame-differencing

$$M(t) = ||B - I(t)|| > \lambda$$

$$B = I(t-1)$$



Adjusting temporal scale of differencing

Note what happens when we adjust the temporal scale (frame rate) at which we perform two-frame differencing …

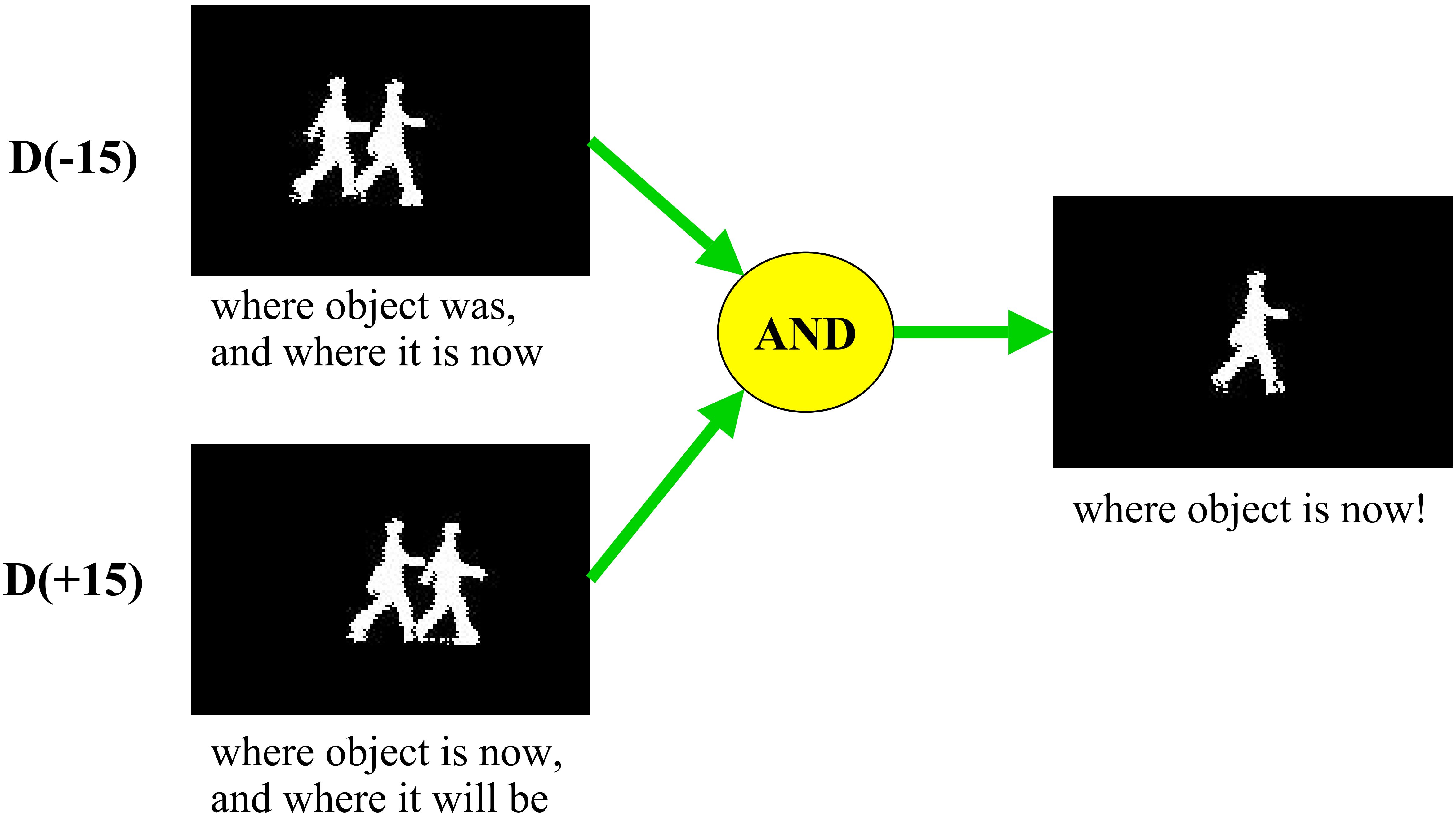
Define D(N) = || I(t) - I(t+N) ||

I(t) D(-1) D(-3) D(-5) D(-9) D(-15)

more complete object silhouette, but two copies (one where object used to be, one where it is now).

A neat “trick”: 3-frame differencing

The previous observation is the motivation behind three-frame differencing



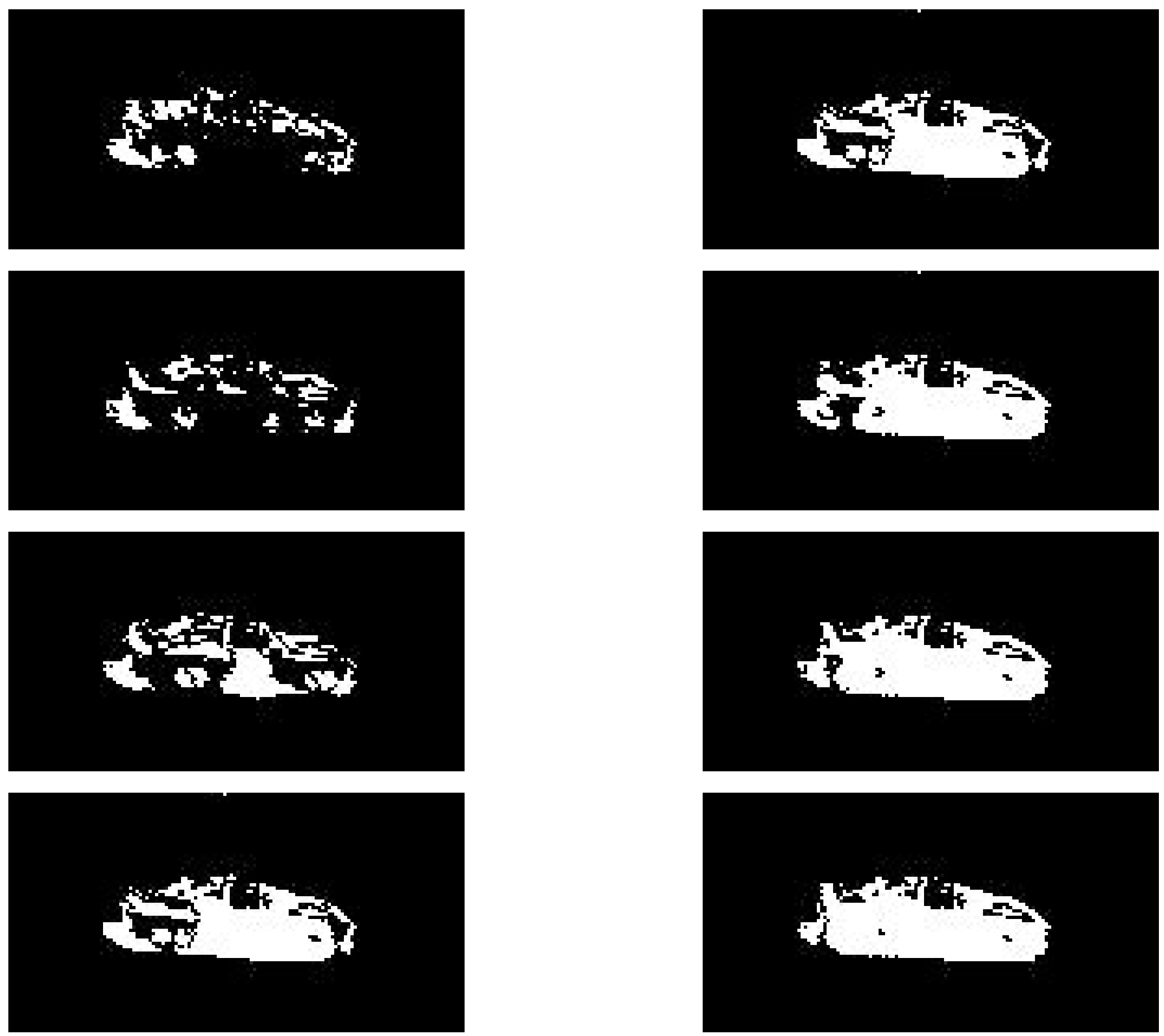
But its hard to find a good frame rate

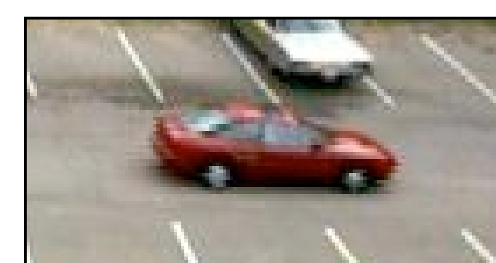
Choice of good frame-rate for three-frame differencing depends on the size and speed of the object

frames skipped

1, 5, 15, 25 | 35, 45, 55, 65

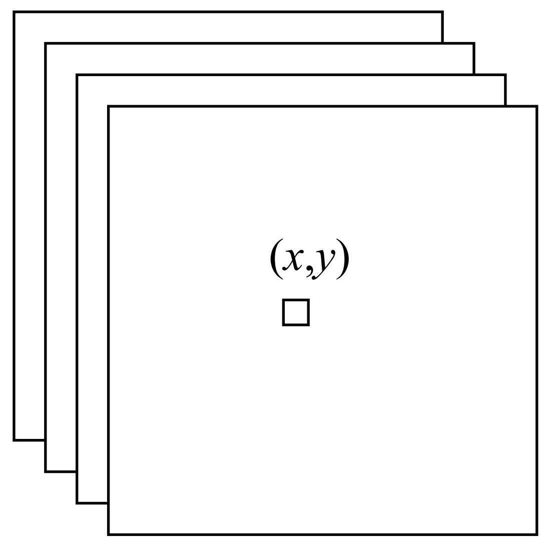
This worked well for the person → 15



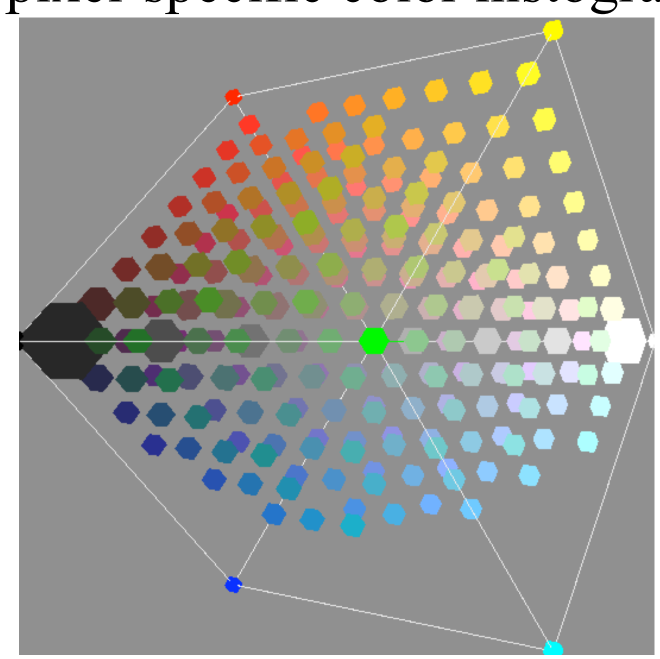


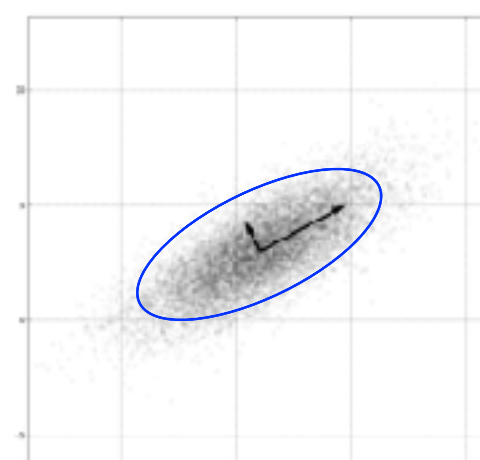
What's a "principled" way to build background model?

Statistical color models: $P(I(x,y)|bg) > \lambda$

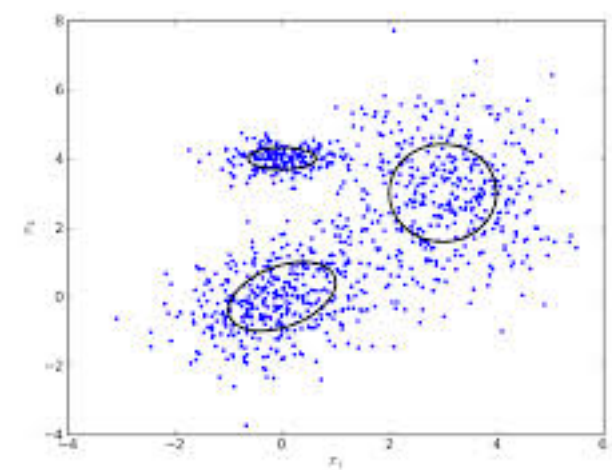


pixel-specific color histogram





$$P(I) = N(I; \mu, \Sigma)$$



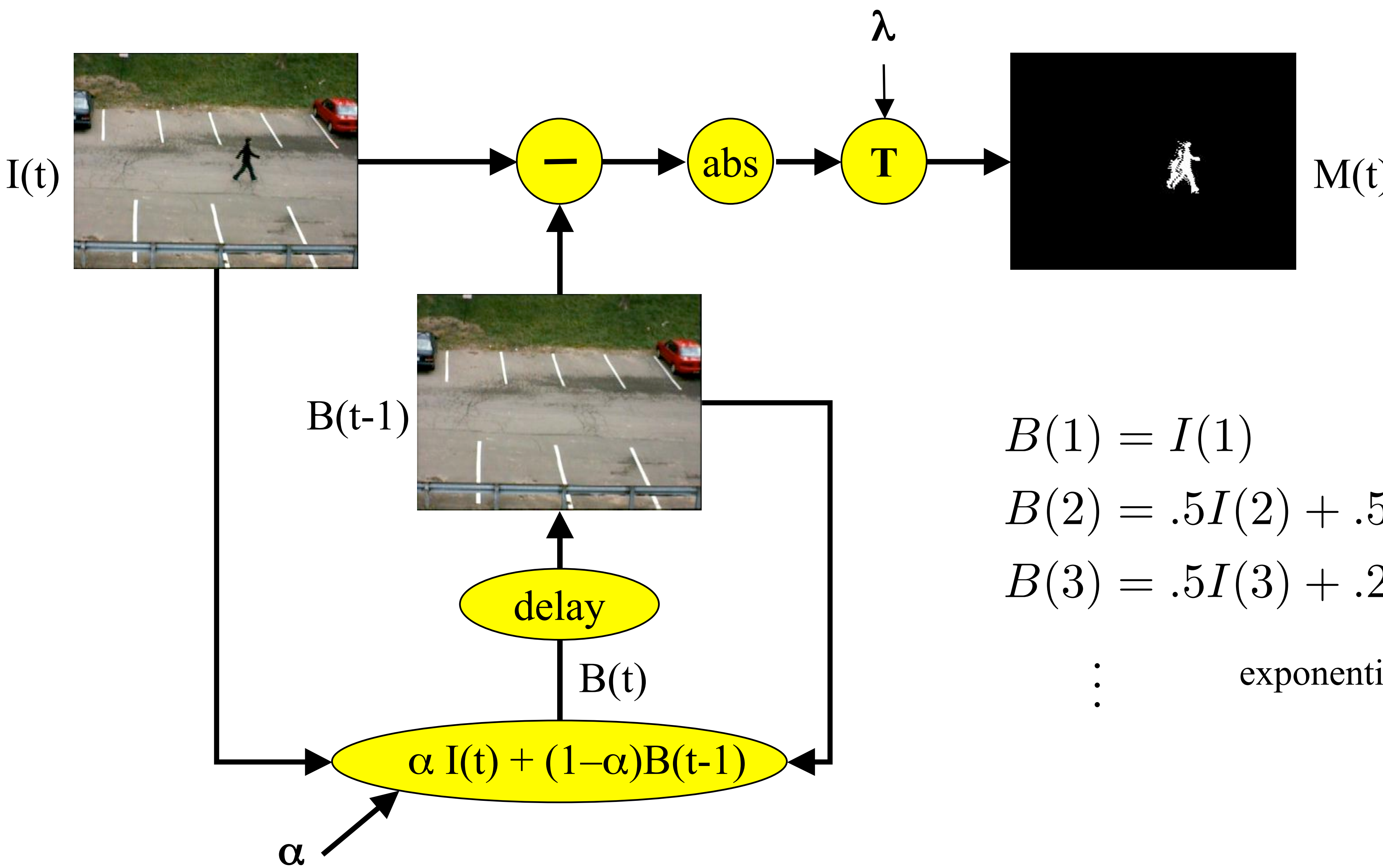
$$P(I) = \sum_i \pi_i N(I; \mu, \Sigma)$$

Online statistical learning (of say, mean)

$$M(t) = [B(t-1) - I(t)] > \lambda$$

$$B(t) = \alpha I(t) + (1-\alpha)B(t-1)$$

alpha=1: frame differencing
alpha=0: fixed (initial) background image



$$B(1) = I(1)$$

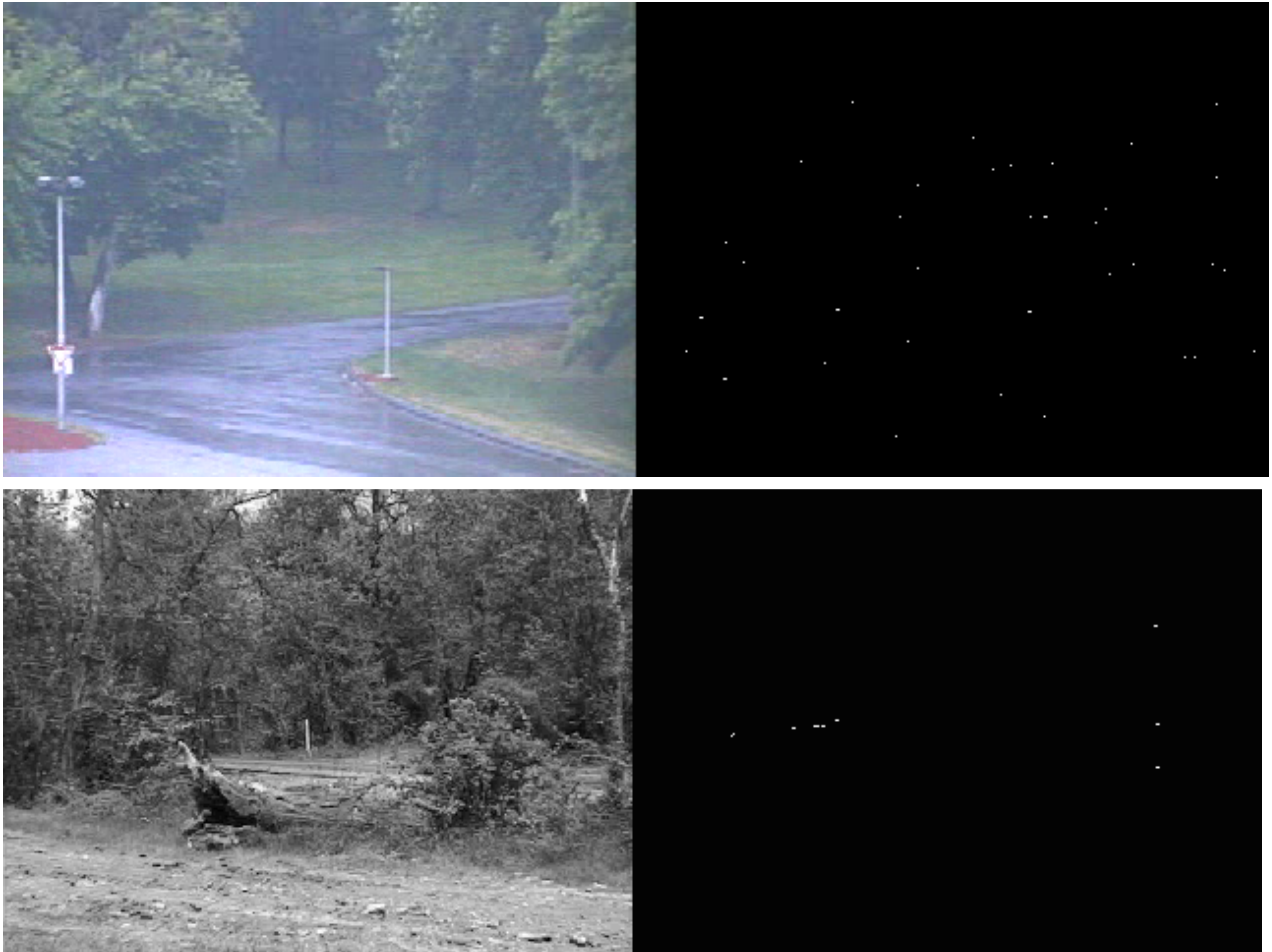
$$B(2) = .5I(2) + .5I(1)$$

$$B(3) = .5I(3) + .25I(2) + .5I(1)$$

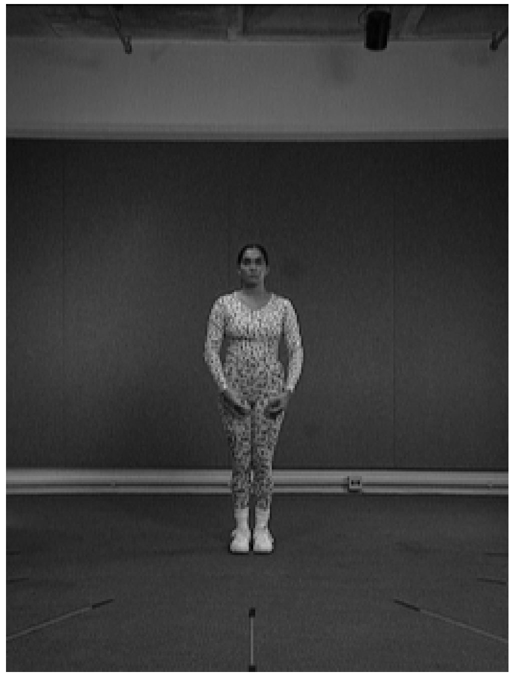
$$\vdots$$

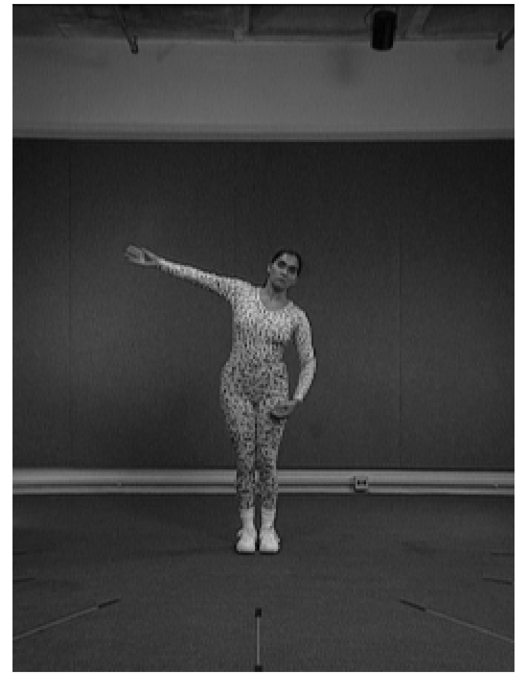
exponential decay

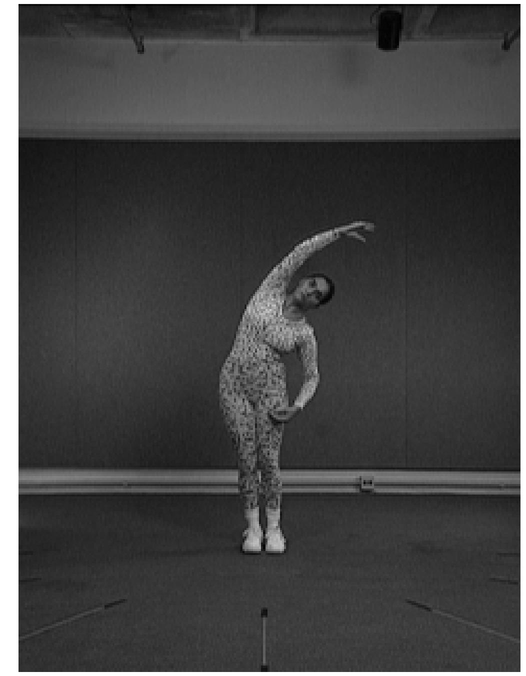
Adaptive background subtraction

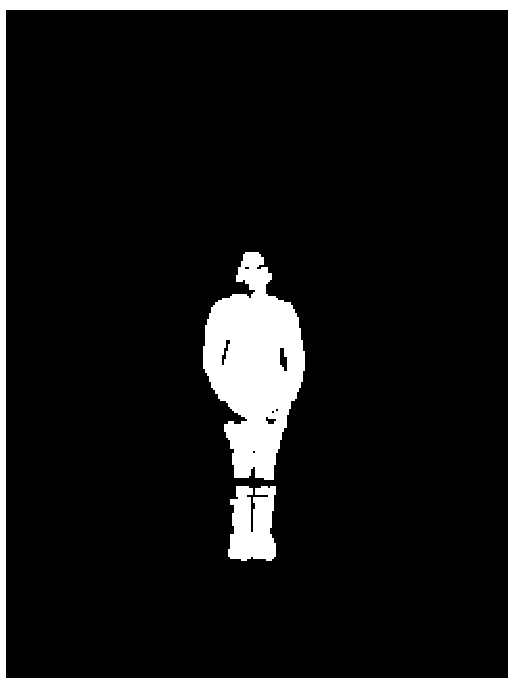


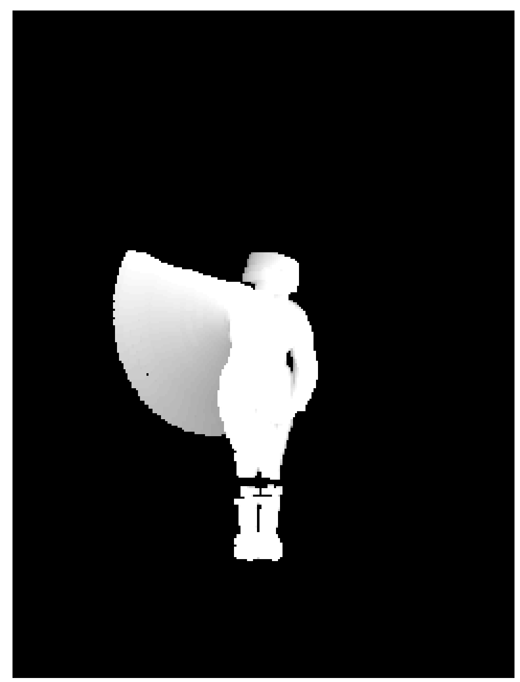
Nifty visualizations: persistant frame differencing

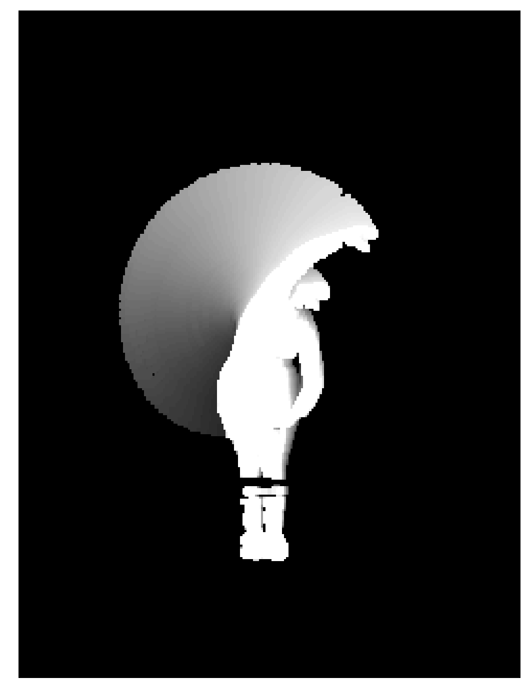

FRAME-0


FRAME-35


FRAME-70


MHI-0


MHI-35


MHI-70

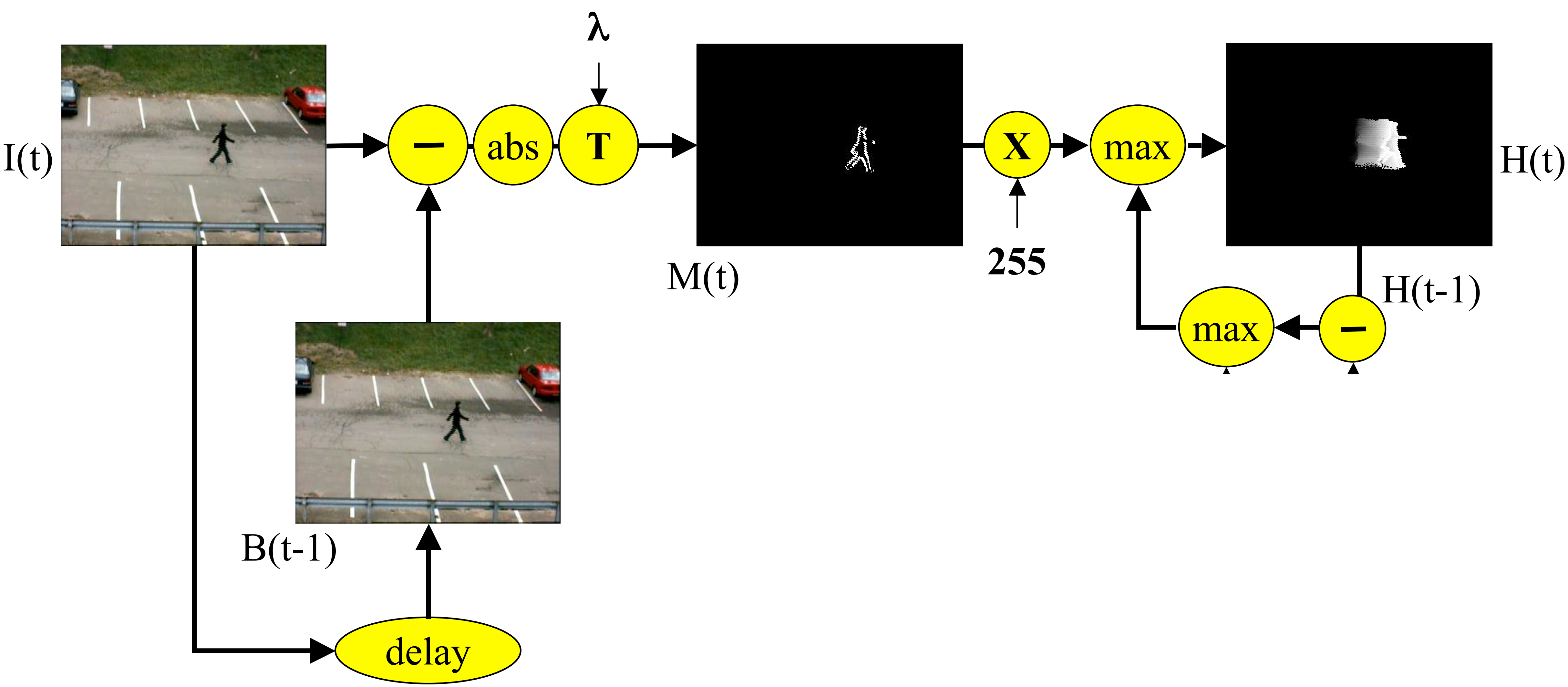
Use some previous method to identify foreground/background pixels

Mark each pixel with the last “time” is was declared foreground

Motion History Images

[Bobick & Davis]

$$H(t) = \max(255 * M(t), \max(H(t) - 1, 0))$$

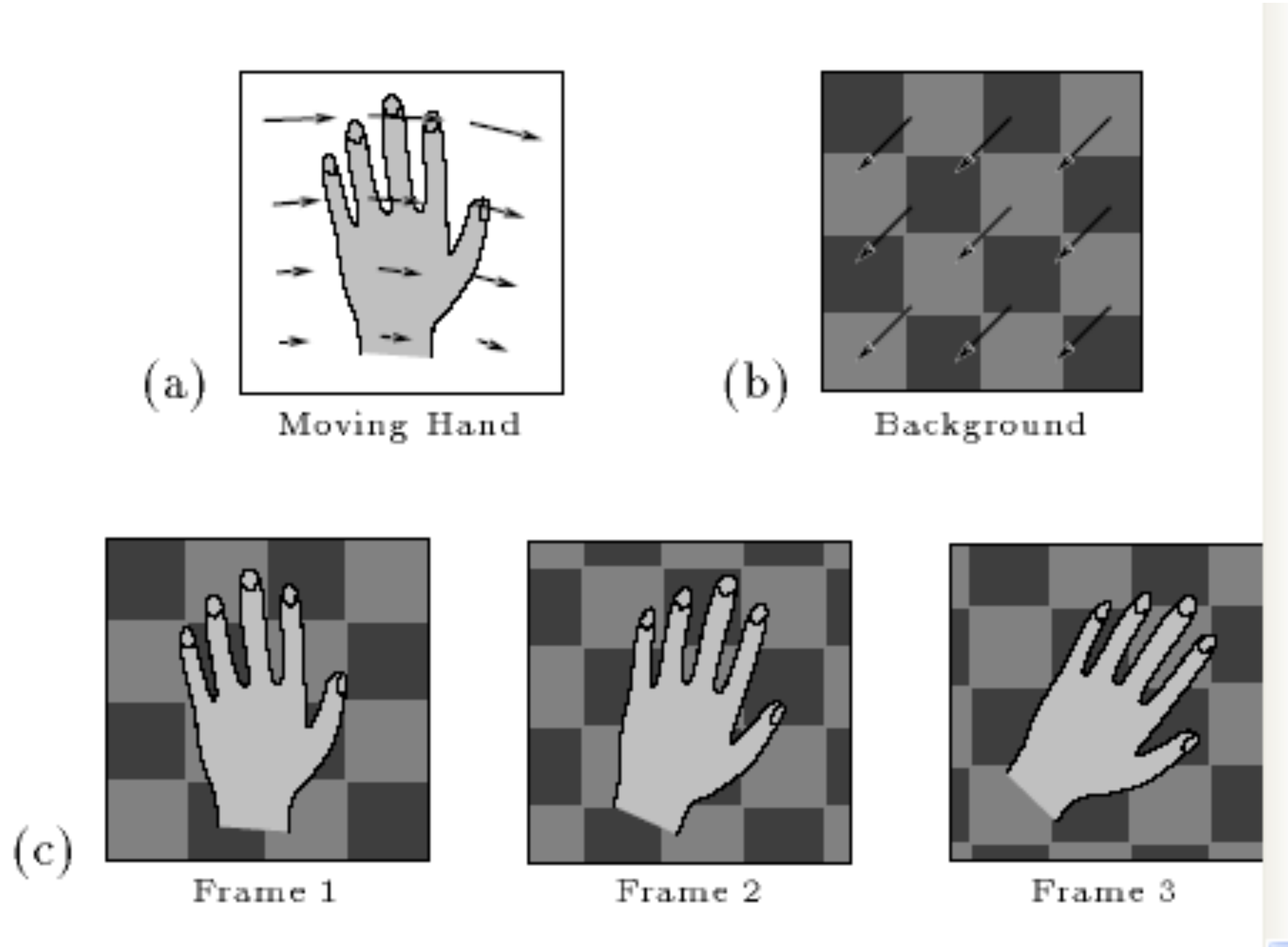


Motion History Images

Outline

- Bightness constancy
- Aperture problem (sparse flow, spatial regularization)
- Small-motion assumption (coarse-to-fine, discrete optimization)
- Motion segmentation (dominant motion estimation, background subtraction, layered models)

Layered model



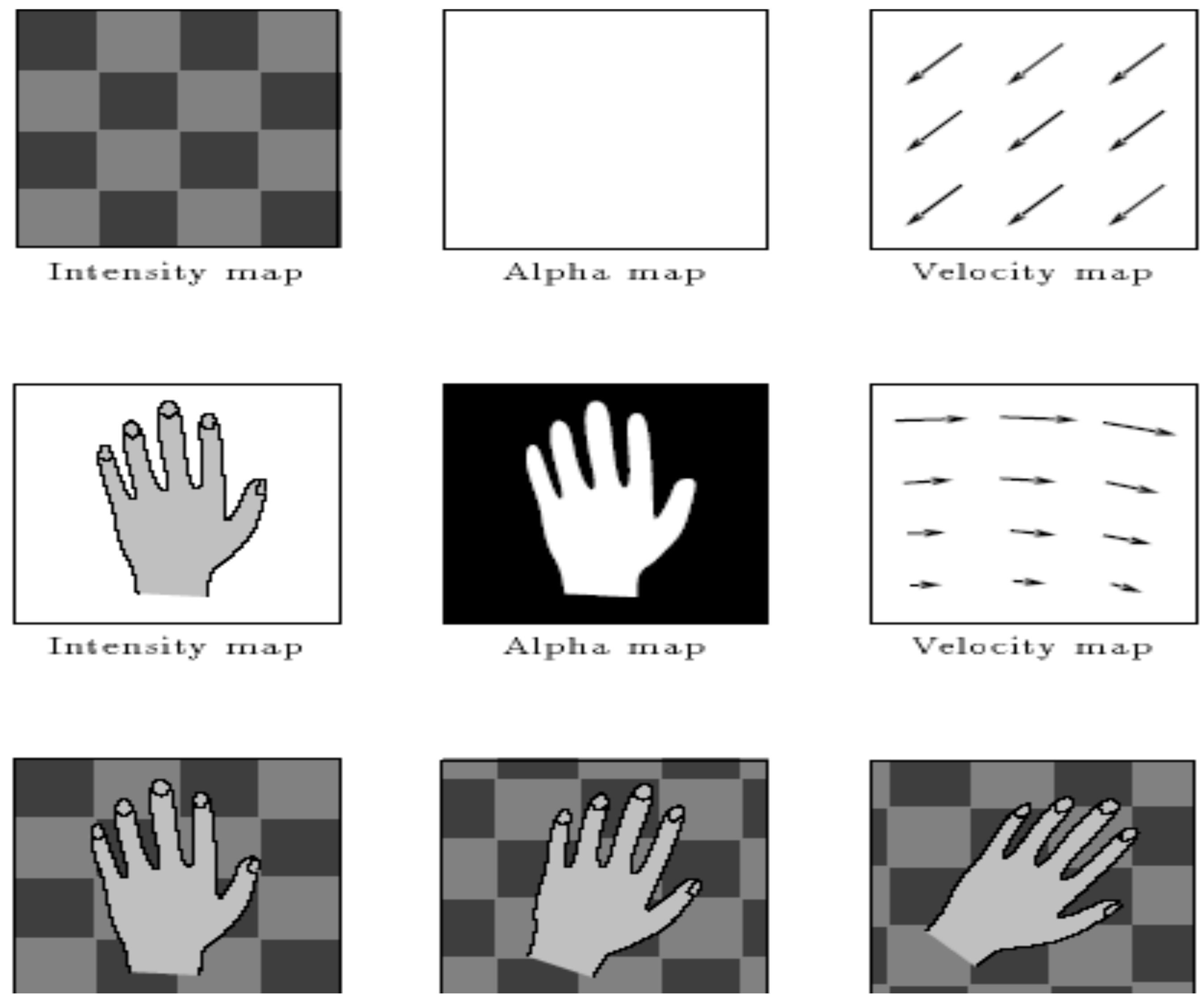
Direct analogy with *layers* in photoshop

Mathematical formalism

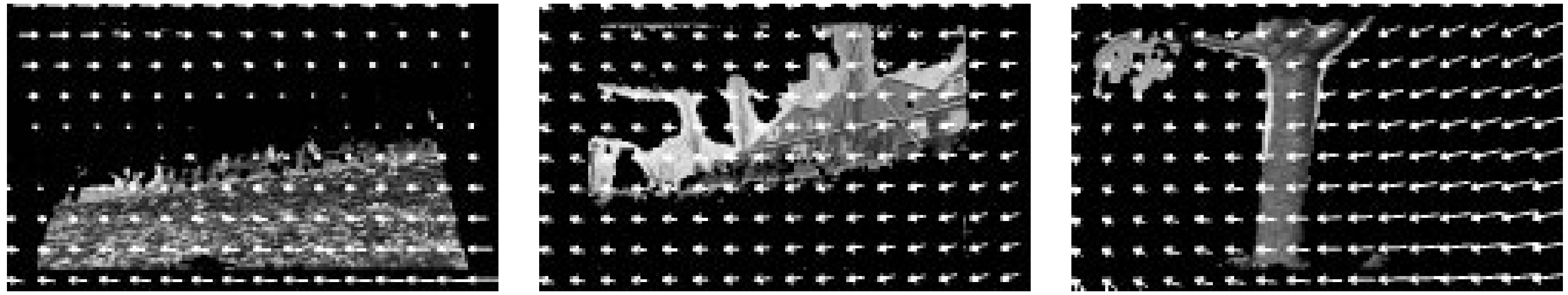
Layer 0 (BG)

Layer 1

Alpha composite



$$I_i(x,y) = \alpha_i(x,y)L_i(x,y) + (1-\alpha_i(x,y))I_{i-1}(x,y)$$



(a) (b) (c)

Figure 12: The layers corresponding to the tree, the flower bed, and the house shown in figures (a-c), respectively. The affine flow field for each layer is superimposed.

(a) (b) (c)

Figure 13: Frames 0, 15, and 30 as reconstructed from the layered representation shown in figures (a-c), respectively.

(a) (b) (c)

Figure 14: The sequence reconstructed without the tree layer shown in figures (a-c), respectively.

Inferring layers, motion, and appearance with EM

Learning Flexible Sprites in Video Layers

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University of Toronto
http://www.psi.toronto.edu

Takeaways

- Bightness constancy
- Aperture problem — (sparse flow, spatial regularization)
- Small-motion assumption — (coarse-to-fine, discrete optimization)
- Motion segmentation — (dominant motion estimation, background subtraction, layered models)