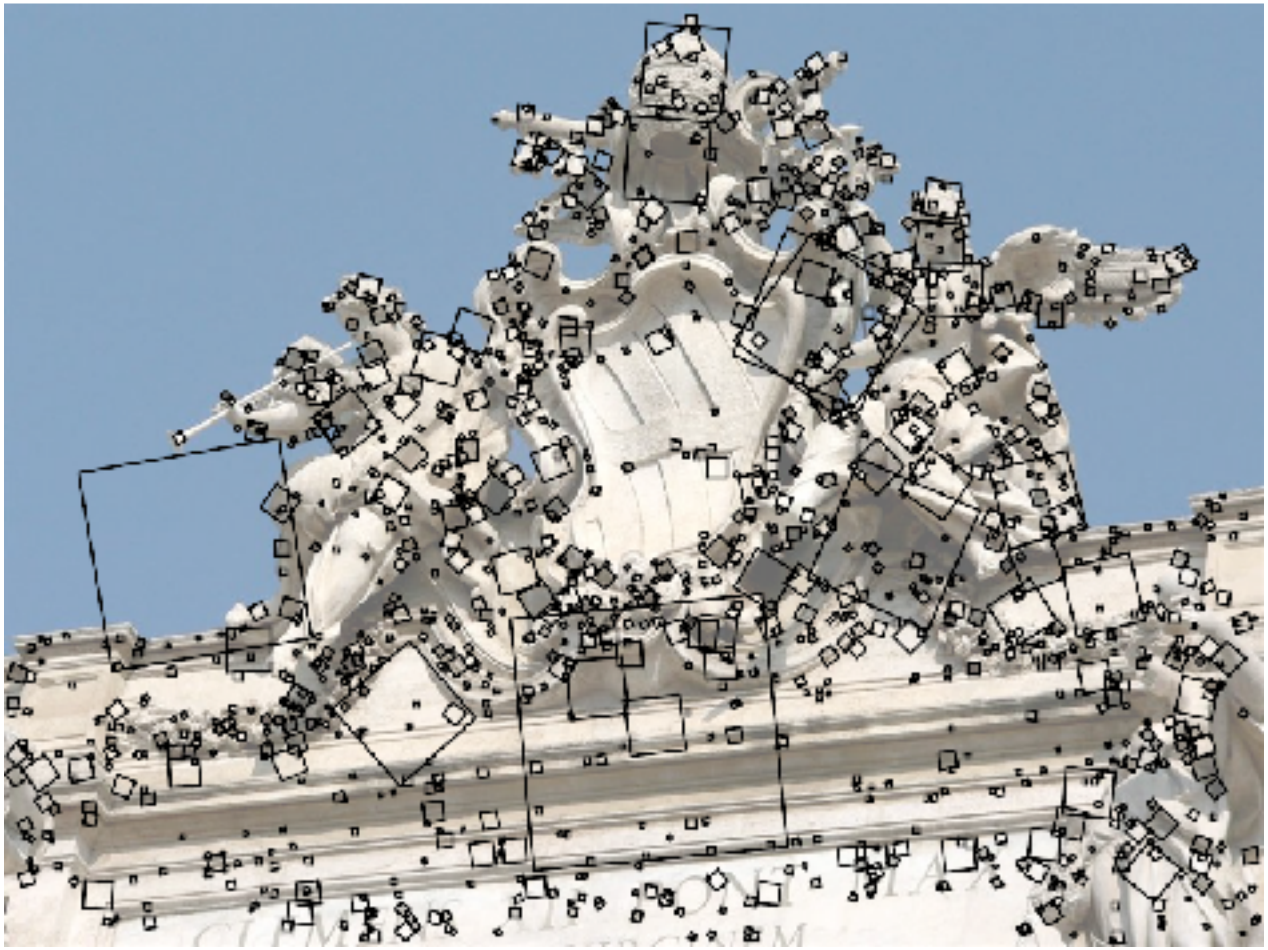
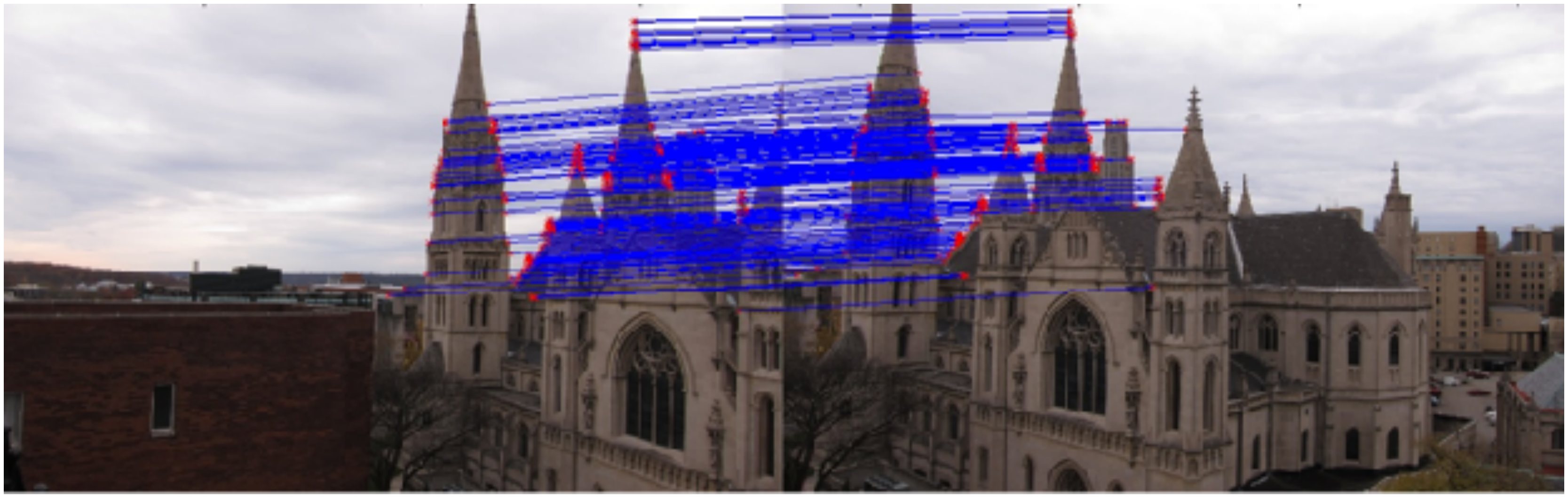
Correspondence

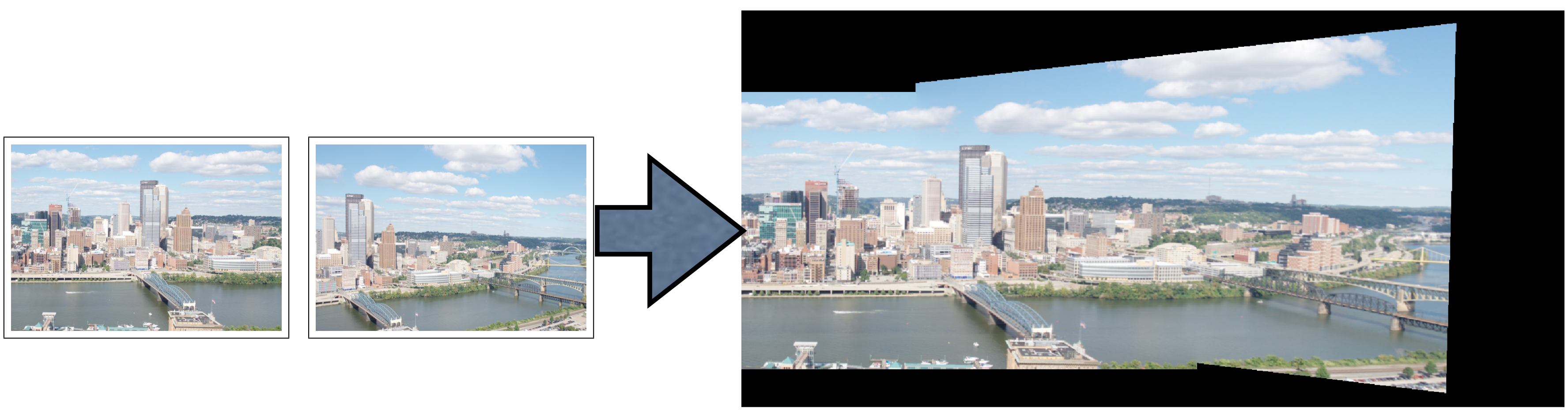


Outline

- Motivation
- Interest point detection
- Descriptors

Core visual understanding task: finding correspondences between images



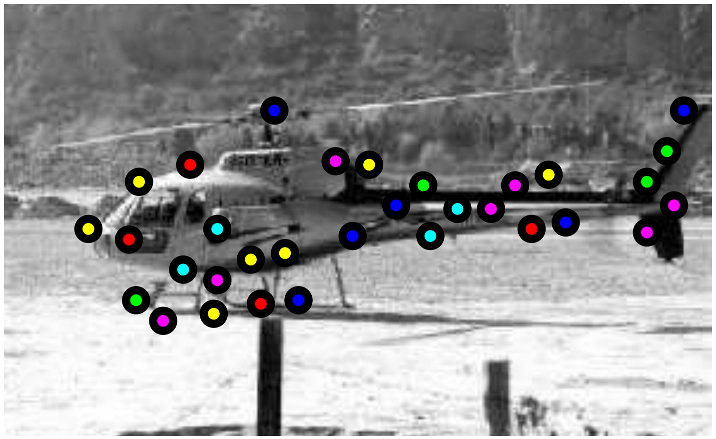


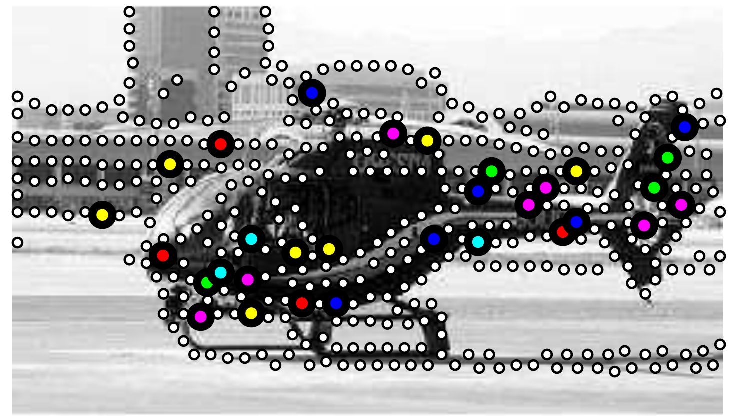
Example: image matching of landmarks

Correspondence + geometry estimation

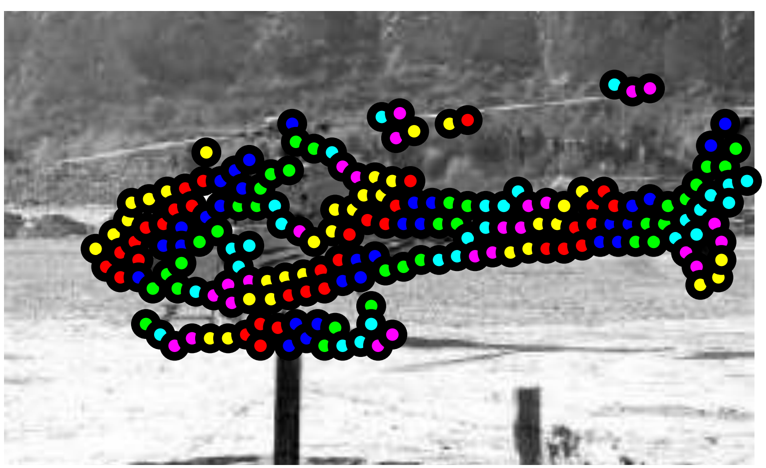
Object recognition by matching

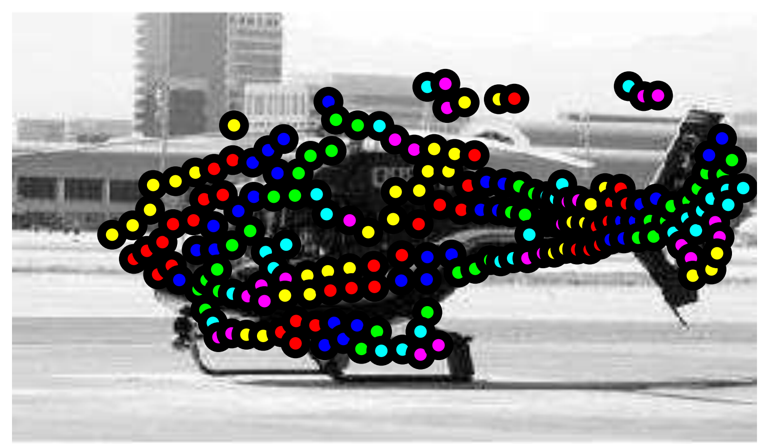
Sparse correspondence





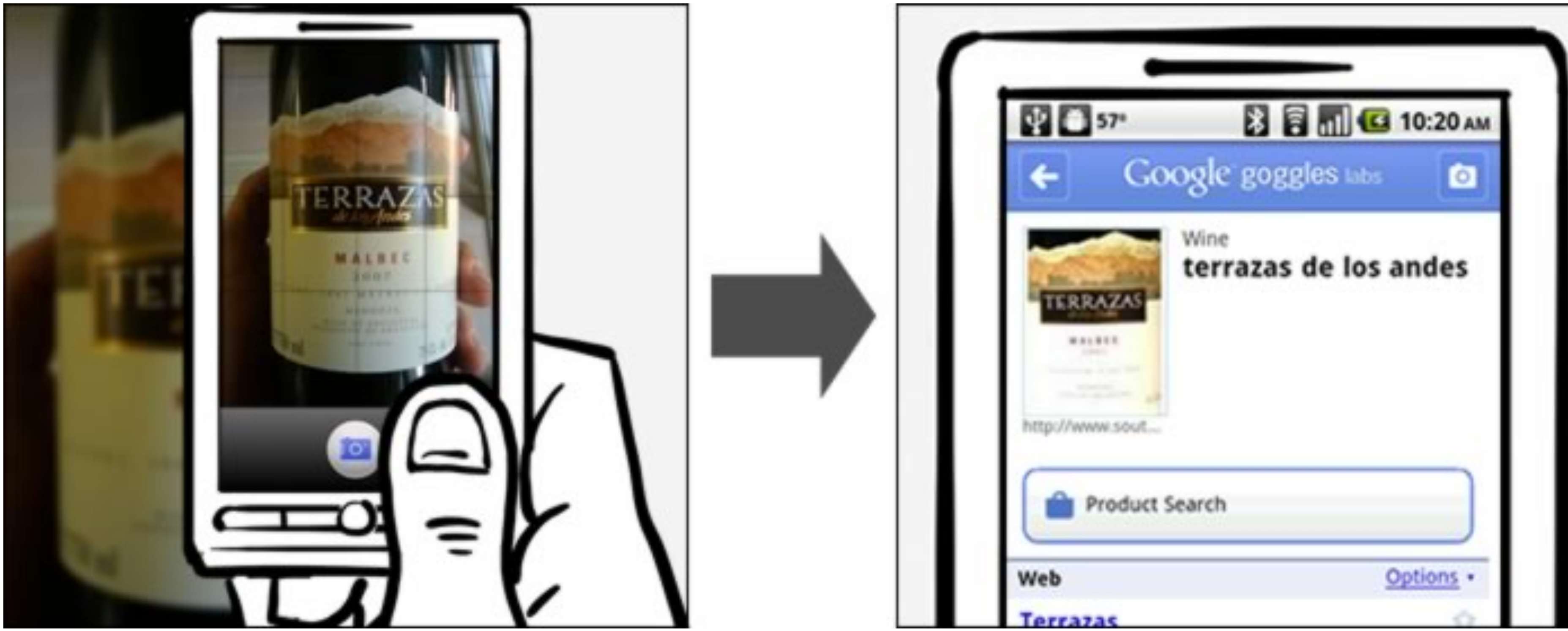
Dense corrrespondence





Example: license plate recognition

Example: product recognition



Google Glass

Reference

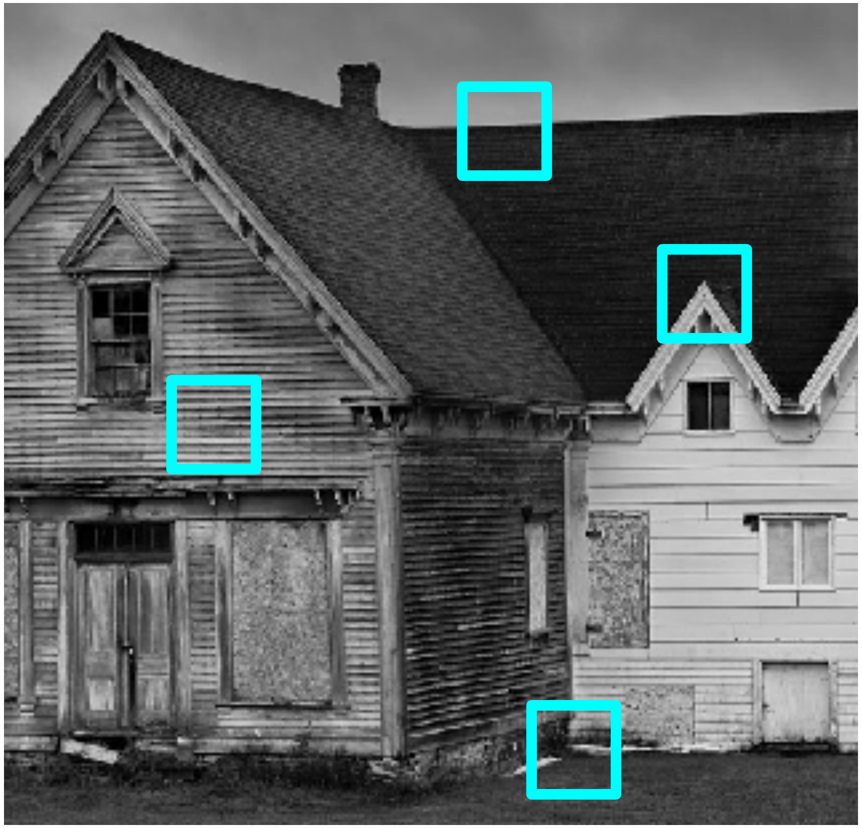
Distinctive Image Features from Scale-Invariant Keypoints

David G. Lowe
Computer Science Department
University of British Columbia
Vancouver, B.C., Canada
lowe@cs.ubc.ca

January 5, 2004

IJCV 04

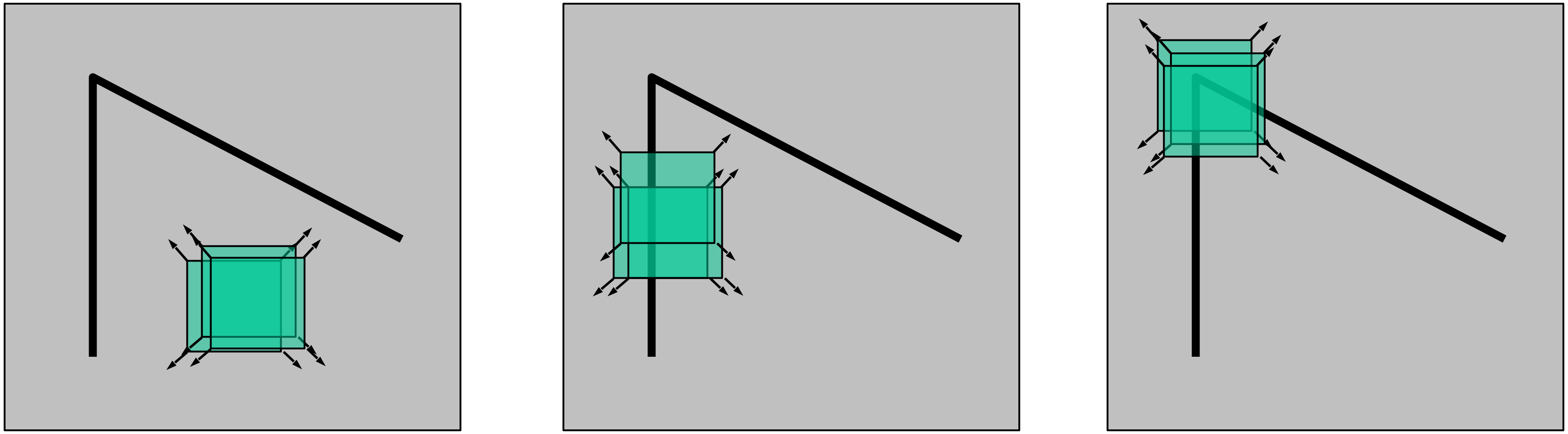
Motivation



Which of these patches are easier to match?

Why? How can we mathematically operationalize this?

Corner Detector: Basic Idea



"flat" region:
no change in any direction

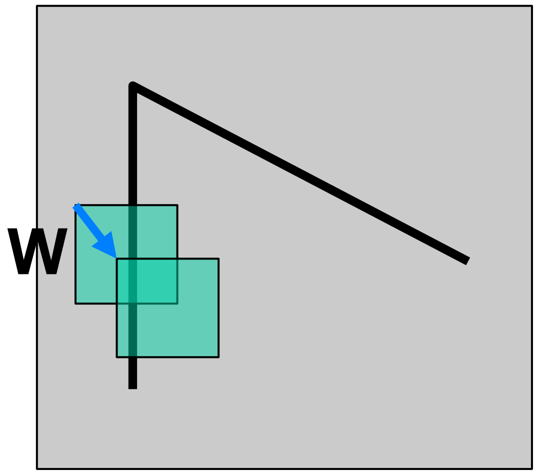
"edge":
no change along the edge direction

"corner":
significant change in all directions

Defn: points are "matchable" if small shifts always produce a large SSD error

The math

Defn: points are "matchable" if small shifts always produce a large SSD error



$$\text{cornerness}(x_0, y_0) = \min_{u,v} E_{x_0,y_0}(u, v)$$

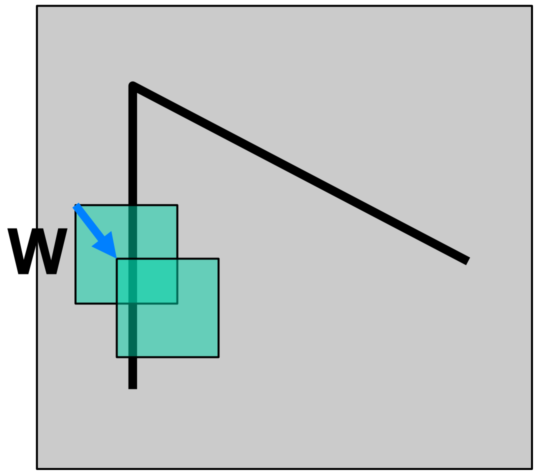
where

$$E_{x_0,y_0}(u, v) = \sum_{(x,y) \in W(x_0,y_0)} [I(x + u, y + v) - I(x, y)]^2$$

Why can't this be right?

The math

Defn: points are “matchable” if small shifts always produce a large SSD error



$$\text{cornerness}(x_0, y_0) = \min_{u^2 + v^2 = 1} E_{x_0,y_0}(u, v)$$

where

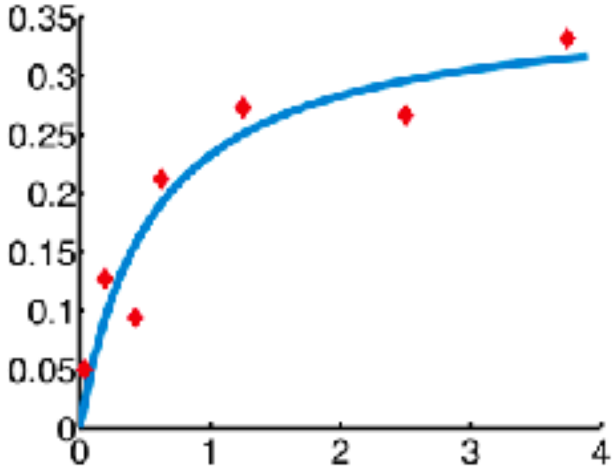
$$E_{x_0,y_0}(u, v) = \sum_{(x,y) \in W(x_0,y_0)} [I(x + u, y + v) - I(x, y)]^2$$

General mathematical tool: nonlinear least squares

$$\text{cornerness}(x_0, y_0) = \min_{u^2 + v^2 = 1} E_{x_0,y_0}(u, v)$$

where

$$E_{x_0,y_0}(u,v) = \sum_{(x,y)\in W(x_0,y_0)} [I(x+u, y+v) - I(x,y)]^2$$

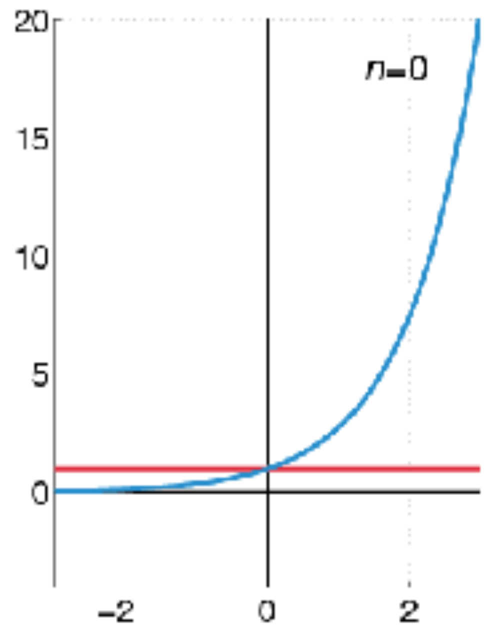


https://en.wikipedia.org/wiki/Non-linear_least_squares

We'll apply a "standard technique": Gauss-Netwon optimization

Background: taylor series expansion

$$f(x+u) = f(x) + \frac{\partial f(x)}{\partial x} u + \frac{1}{2} \frac{\partial f(x)}{\partial x x} u^2 + \text{Higher Order Terms}$$



Approximation of $f(x) = e^x$ at x=0

Why are low-order expansions reasonable?
Underyling smoothness of real-world signals

Multivariate taylor series

$$I(x+u, y+v) = I(x,y) + \begin{bmatrix} \frac{\partial I(x,y)}{\partial x} & \frac{\partial I(x,y)}{\partial y} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} +$$

what's this vector called?

$$\frac{1}{2} \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} \frac{\partial I(x,y)}{\partial xx} & \frac{\partial I(x,y)}{\partial xy} \\ \frac{\partial I(x,y)}{\partial xy} & \frac{\partial I(x,y)}{\partial yy} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} + \text{Higher Order Terms}$$

what's this matrix called?

$$I(x+u, y+v) \approx \mathbf{I} + \mathbf{I}_x u + \mathbf{I}_y v$$

where

$$\mathbf{I}_x = \frac{\partial I(x,y)}{\partial x}$$

Multivariate taylor series

$$I(x+u,y+v) = I(x,y) + \underbrace{\begin{bmatrix} \frac{\partial I(x,y)}{\partial x} & \frac{\partial I(x,y)}{\partial y} \end{bmatrix}}_{\text{gradient}} \begin{bmatrix} u \\ v \end{bmatrix} +$$

$$\frac{1}{2} \begin{bmatrix} u & v \end{bmatrix} \underbrace{\begin{bmatrix} \frac{\partial I(x,y)}{\partial xx} & \frac{\partial I(x,y)}{\partial xy} \\ \frac{\partial I(x,y)}{\partial xy} & \frac{\partial I(x,y)}{\partial yy} \end{bmatrix}}_{\text{Hessian}} \begin{bmatrix} u \\ v \end{bmatrix} + \text{Higher Order Terms}$$

$$I(x+u,y+v) \approx \mathbf{I} + \mathbf{I}_x u + \mathbf{I}_y v$$

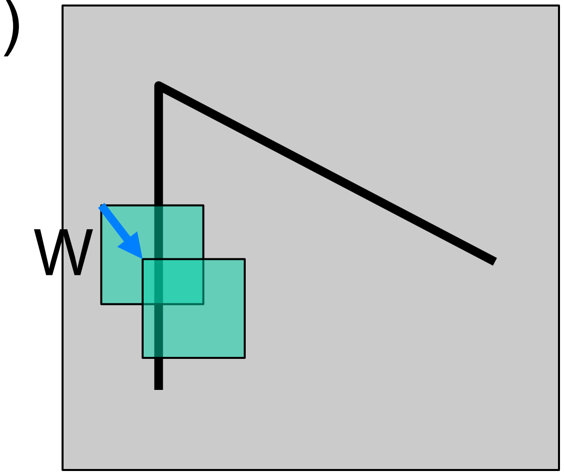
where

$$\mathbf{I}_x = \frac{\partial I(x,y)}{\partial x}$$

Feature detection: the math

Consider shifting the window W by (u,v)

- how do the pixels in W change?
- compare each pixel before and after by summing up the squared differences
- this defines an "error" of E(u,v):



$$E(u,v) = \sum_{(x,y)\in W} [I(x+u, y+u) - I(x,y)]^2$$

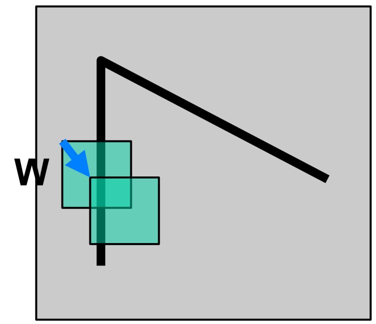
$$\approx \sum_{(x,y)\in W} [\mathbf{I} + \mathbf{I}_x u + \mathbf{I}_y v - \mathbf{I}]^2$$

$$= \sum_{(x,y)\in W} [\mathbf{I}_x^2 u^2 + \mathbf{I}_y^2 v^2 + 2\mathbf{I}_x \mathbf{I}_y uv]$$

$$= \begin{bmatrix} u & v \end{bmatrix} A \begin{bmatrix} u \\ v \end{bmatrix}, \quad A = \sum_{(x,y)\in W} \begin{bmatrix} \mathbf{I}_x^2 & \mathbf{I}_x \mathbf{I}_y \\ \mathbf{I}_y \mathbf{I}_x & \mathbf{I}_y^2 \end{bmatrix}$$

The math (cont'd)

Defn: points are "matchable" if small shifts always produce a large SSD error



$$\text{Corner}(x_0, y_0) = \min_{u^2+v^2=1} E(u, v)$$

where

second-moment matrix
is simple products of $\mathbf{I}_x$,$\mathbf{I}_y$

$$E(u,v) = \begin{bmatrix} u & v \end{bmatrix} A \begin{bmatrix} u \\ v \end{bmatrix}, \quad A = \sum_{(x,y) \in W(x_0,y_0)} \begin{bmatrix} \mathbf{I}_x^2 & \mathbf{I}_x \mathbf{I}_y \\ \mathbf{I}_y \mathbf{I}_x & \mathbf{I}_y^2 \end{bmatrix}$$

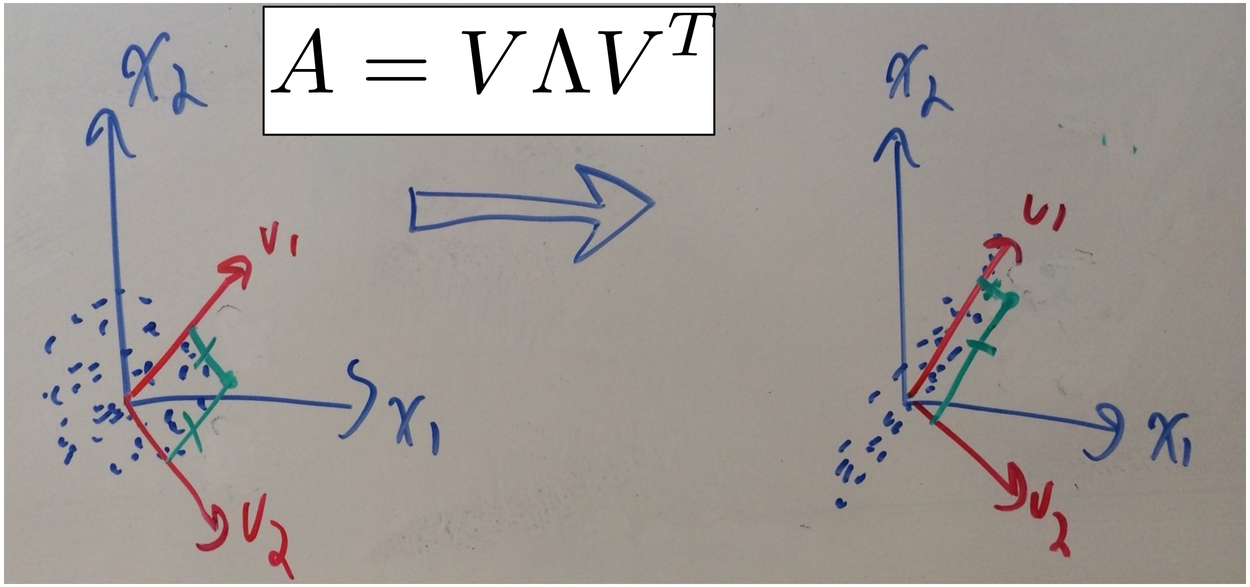
Claim 1: 'A' is symmetric ($A^T = A$) and PSD

Claim 2: Corner-ness is given by min eigenvalue of 'A'

Question: Is 'A' a Hessian matrix?

Recall: spectral decompositions

$$A = V\Lambda V^T$$



Defn: a symmetric matrix A is PSD if $x^T Ax$ >=0 for all x

A is PSD <=> eigenvalues are all positive

A is PSD <=> A = XX^T, where $X = [\sqrt{\lambda_1}v_1 \quad \sqrt{\lambda_2}v_2 \ldots] = \Lambda^{\frac{1}{2}}V$

$$X = U\Sigma V^T \rightarrow A = XX^T = U\Sigma^2 U^T$$

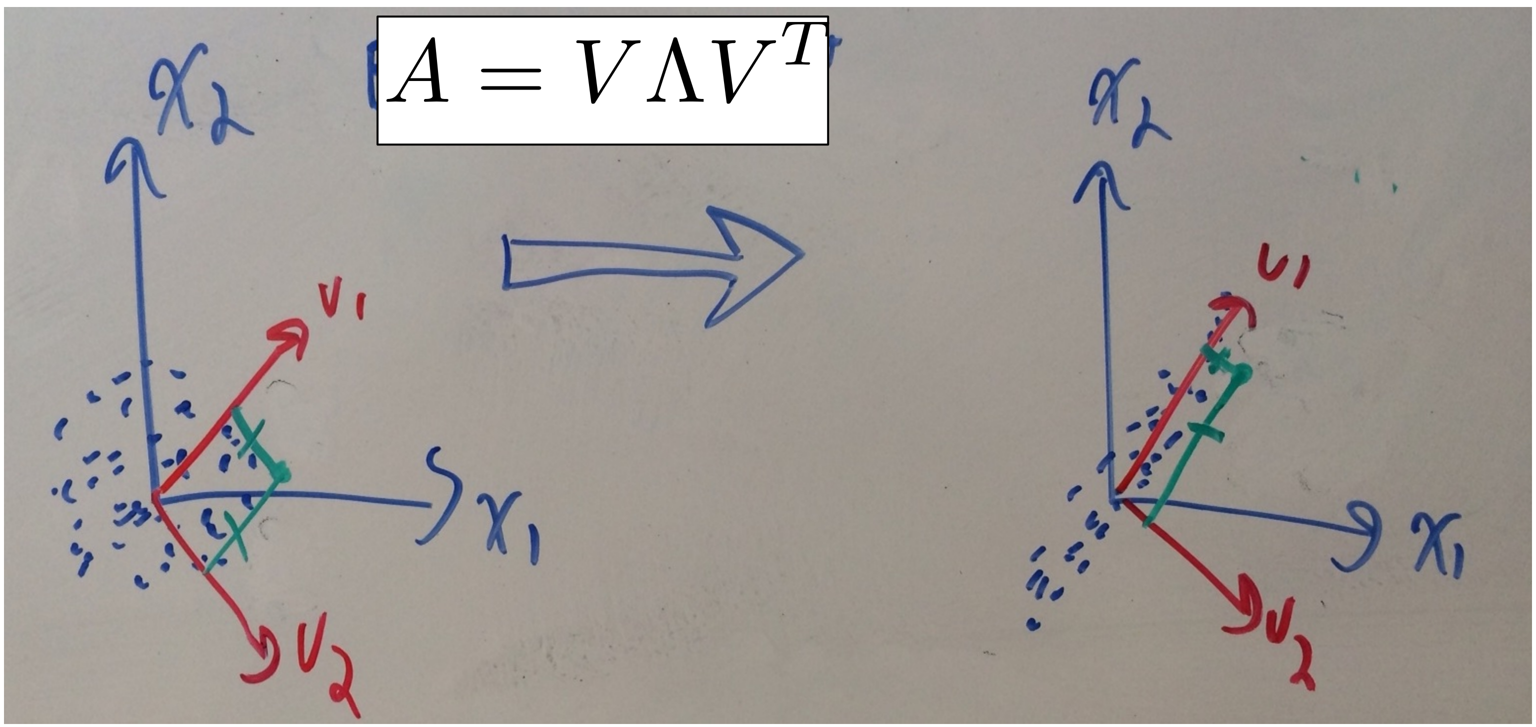
Eigenvectors of A = left singular vectors of X

Eigenvalues of A = squared singular values of X

Aside: turns out spectral decomposition holds for *any* symmetric matrix

$$A = V \Lambda V^T$$

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 & 0 \dots \\ 0 & \lambda_2 & 0 \dots \\ 0 & 0 & \lambda_3 \dots \\ \vdots & \vdots & \vdots \end{bmatrix} \qquad V = \begin{bmatrix} v_1 & v_2, \dots v_n \end{bmatrix} \qquad V^T V = I$$



In the general case, eigenvalues can be negative

Alternative visualization of PSD matrices

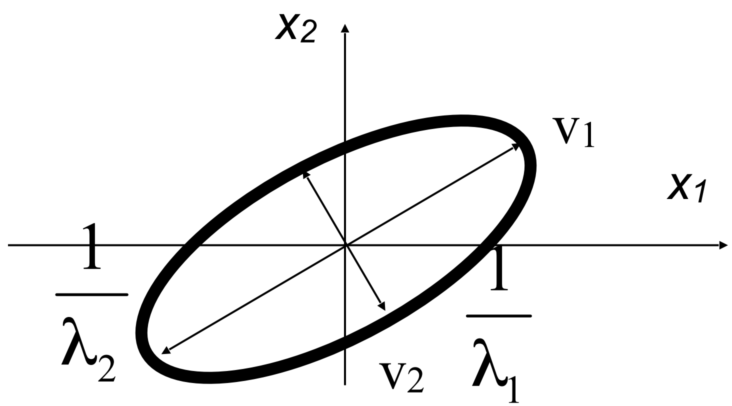
$$A = V \Lambda V^T$$

Consider set of (x_1,x_2) points for which: $\begin{bmatrix} x_1 & x_2 \end{bmatrix} A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 1$

$$A = I \qquad x_1^2 + x_2^2 = 1 \rightarrow (1,0)(0,1)$$

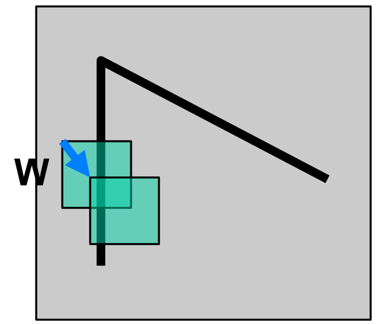
$$A = \Lambda \qquad 4x_1^2 + x_2^2 = 1 \rightarrow (.5,0)(0,1)$$

$$A = V \Lambda V^T$$



Back to corner(ness)

Defn: points are “matchable” if small shifts always produce a large SSD error



$$\text{Corner}(x_0, y_0) = \min_{u^2+v^2=1} E(u,v)$$

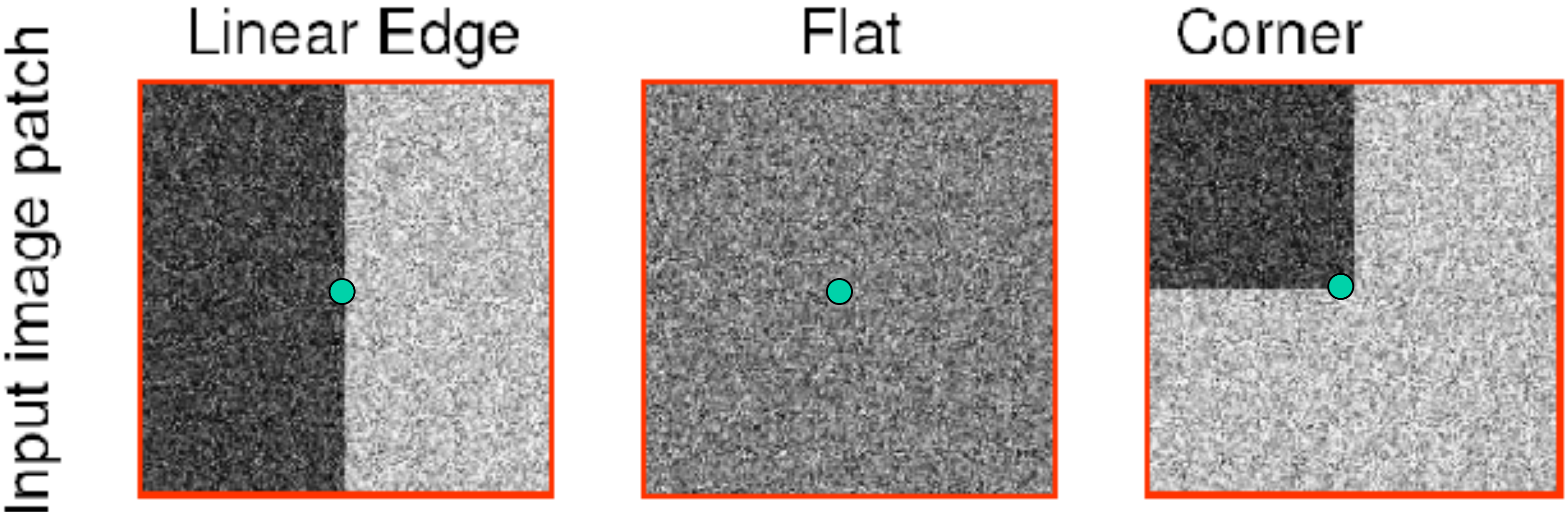
where

$$E(u,v) = \begin{bmatrix} u & v \end{bmatrix} A \begin{bmatrix} u \\ v \end{bmatrix}, \quad A = \sum_{(x,y) \in W(x_0,y_0)} \begin{bmatrix} \mathbf{I}_x^2 & \mathbf{I}_x \mathbf{I}_y \\ \mathbf{I}_y \mathbf{I}_x & \mathbf{I}_y^2 \end{bmatrix}$$

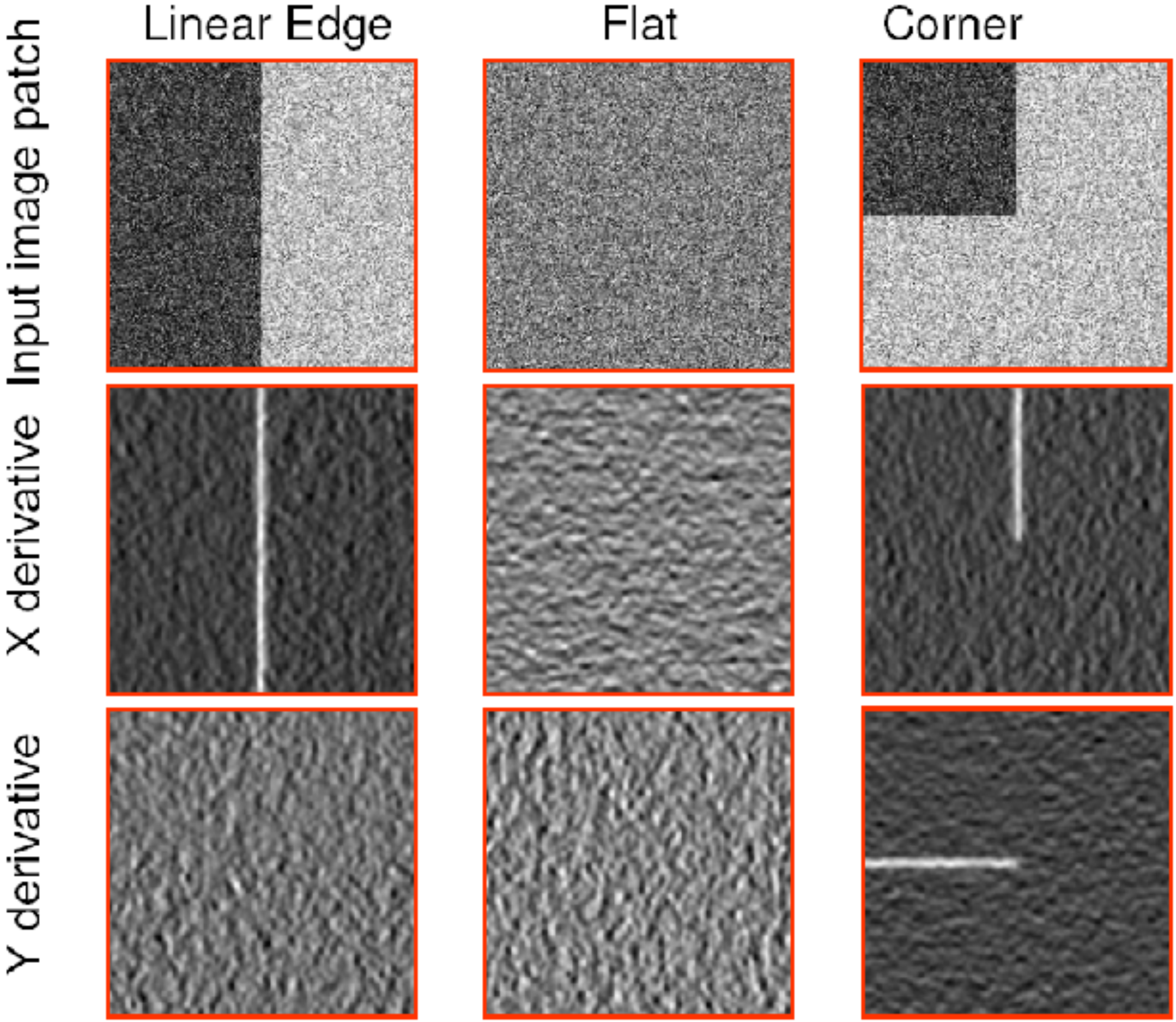
Solution is given by minimum eigenvalue
Implies (xo,yo) is a good corner if minimum eigenvalue is large

(or alternatively, if *both* eigenvalues of ‘A’ are large)

What will eigenvalues (and eigenvectors) look like?

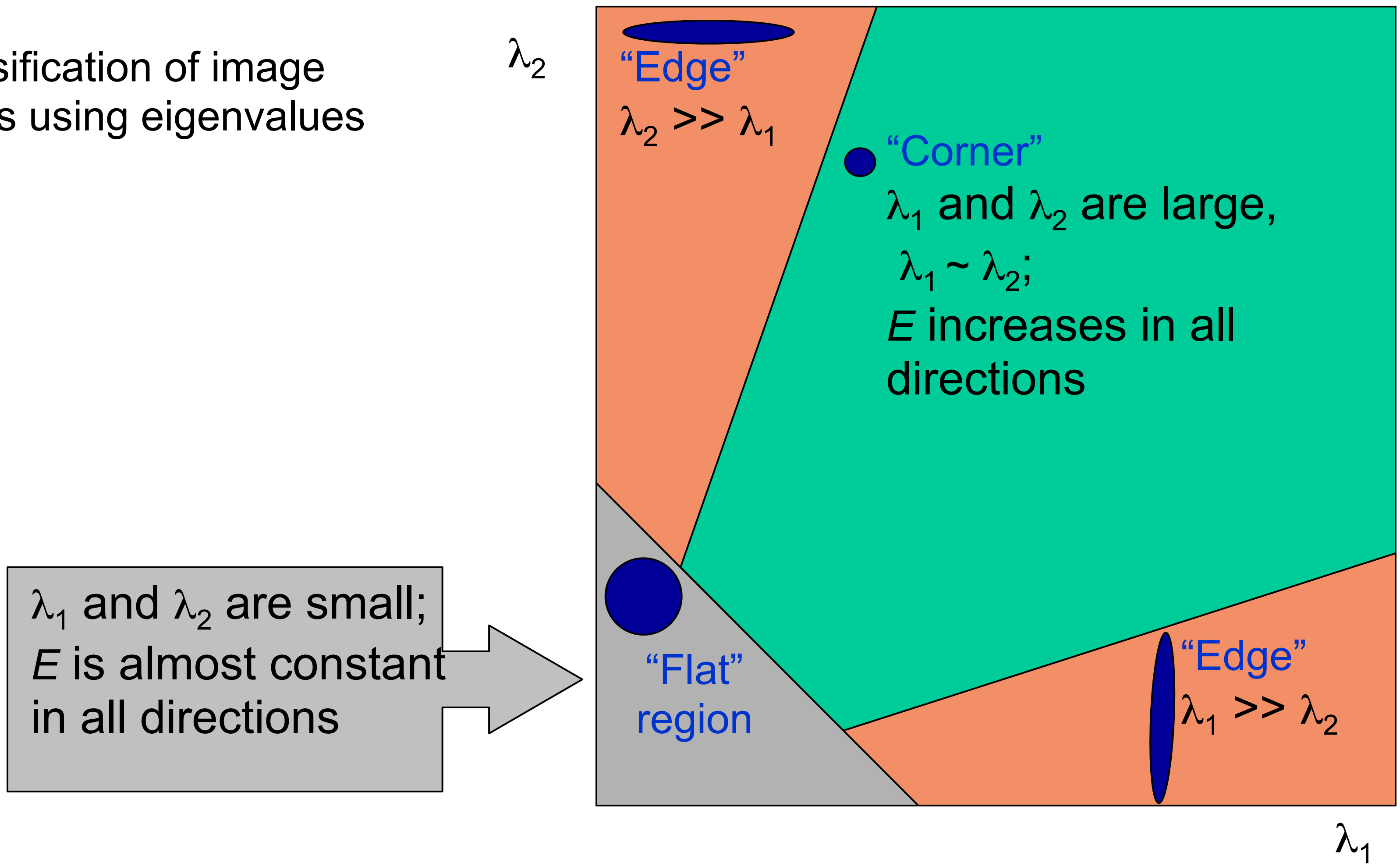


let's think about 'A' matrix…


Linear Edge
Flat
Corner
Input image patch
X derivative
Y derivative

Intuition behind eigenvalues

Classification of image points using eigenvalues of *A*:



Efficient computation

Computing eigenvalues (and eigenvectors) is expensive

Turns out that it's easy to compute their sum (trace) and product (determinant)

– $\mathrm{Det}(A) = \lambda_{\mathrm{min}}\lambda_{\mathrm{max}}$
– $\mathrm{Trace}(A) = \lambda_{\mathrm{min}}+\lambda_{\mathrm{max}}$ (trace = sum of diagonal entries)

$$R = 4\frac{Det(A)}{Trace(A)^2}$$

(is proportional to the ratio of eigvenvalues and is 1 if they are equal)

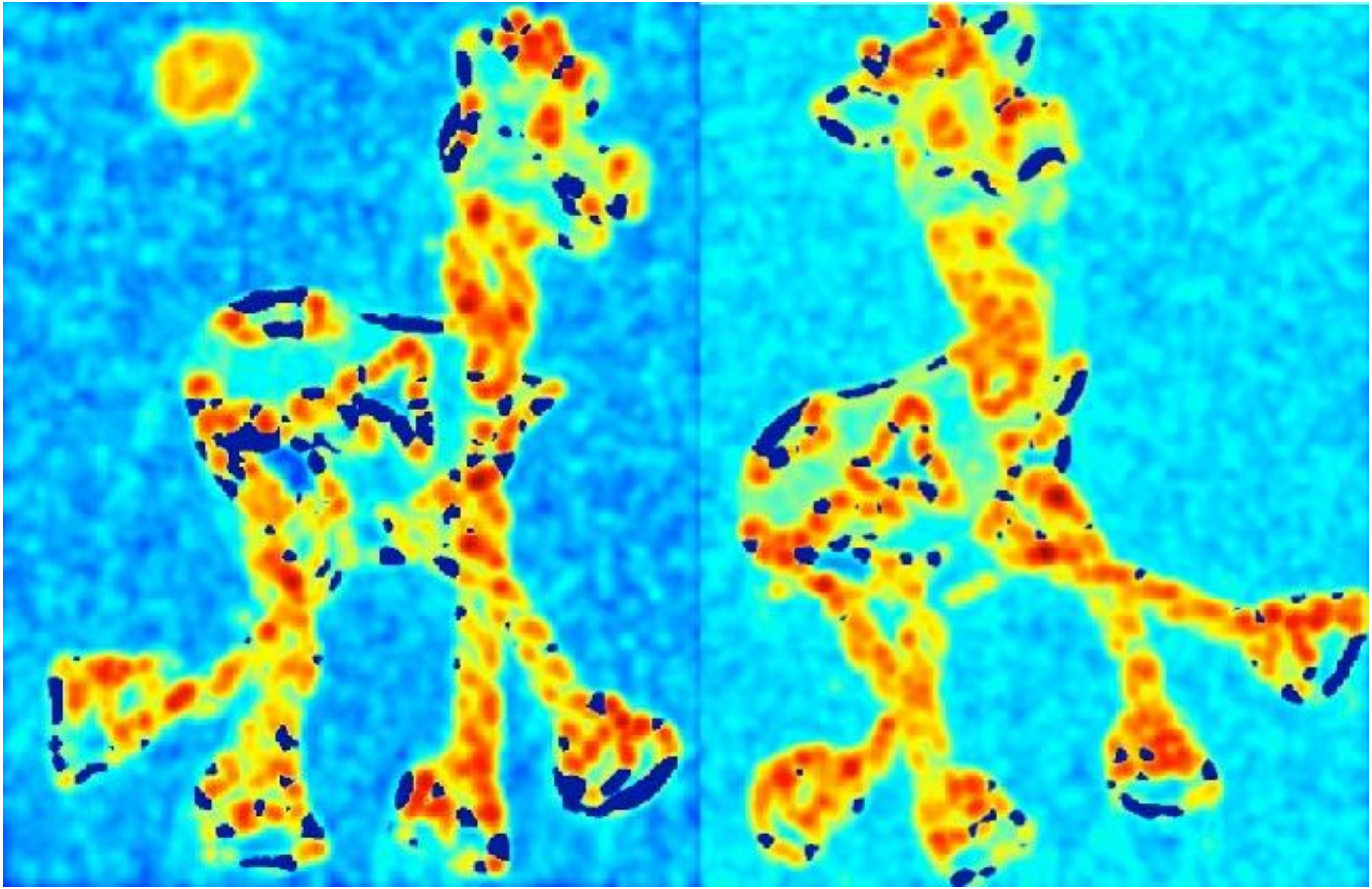
$$R = Det(A) - \alpha Trace(A)^2$$

(also favors large eigenvalues)

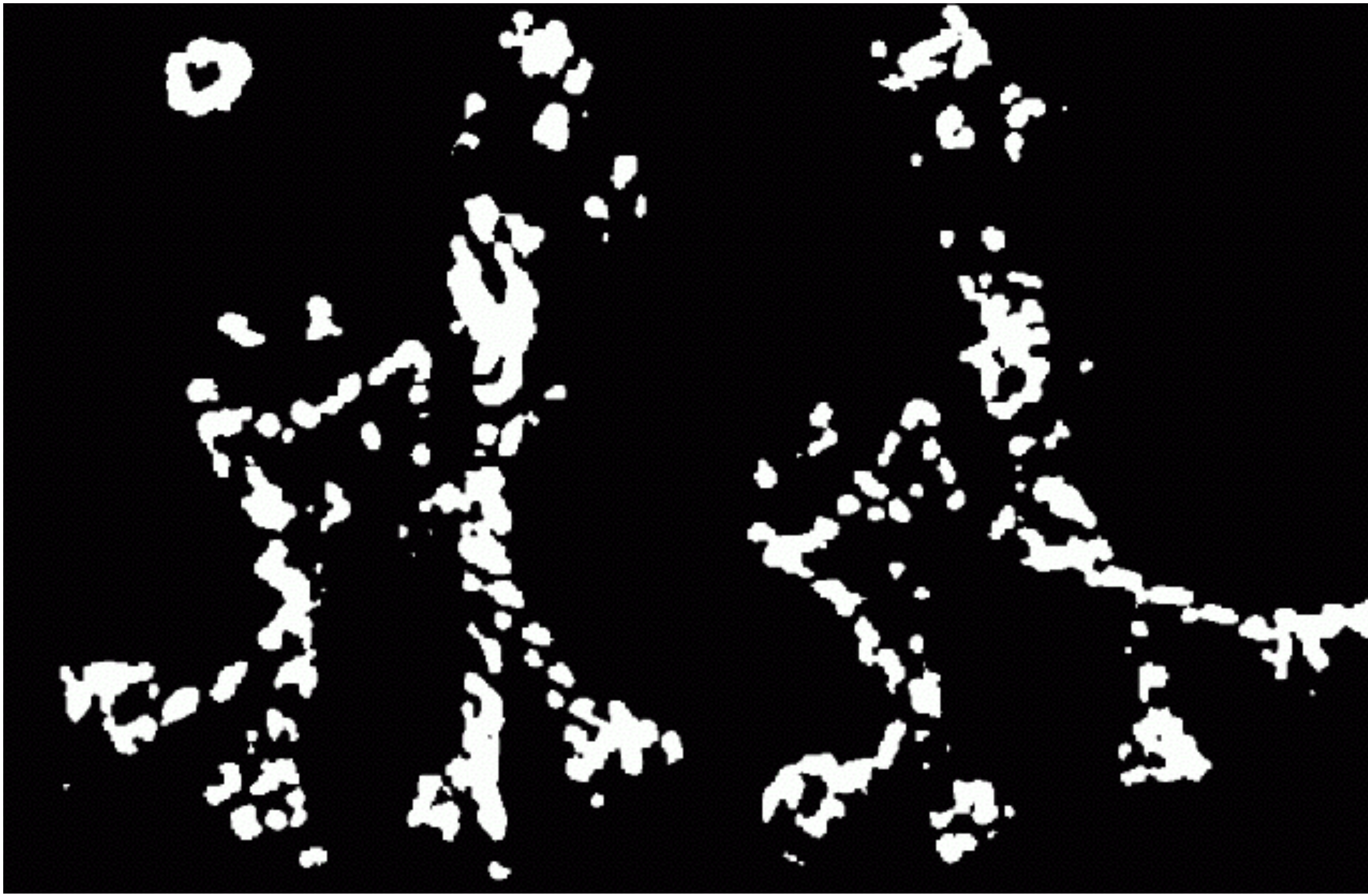
Harris detector example

corner value (red high, blue low)

Question: can we compute these heat maps with convolutions?



Threshold (f > value)



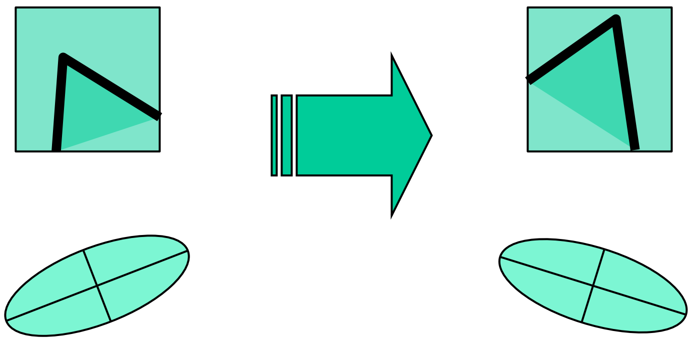
Harris features (in red)

The tops of the horns are detected in both images

Scale and rotation invariance

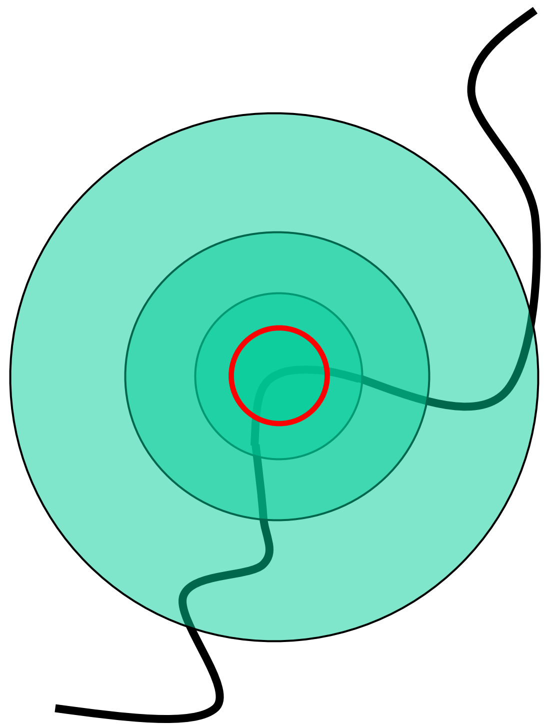
Will interest point detector still fire on rotated & scaled images?

Rotation invariance (?)



Are eigenvector stable under rotations? No
Are eigenvalues stable under rotations? Yes

Scale invariance?



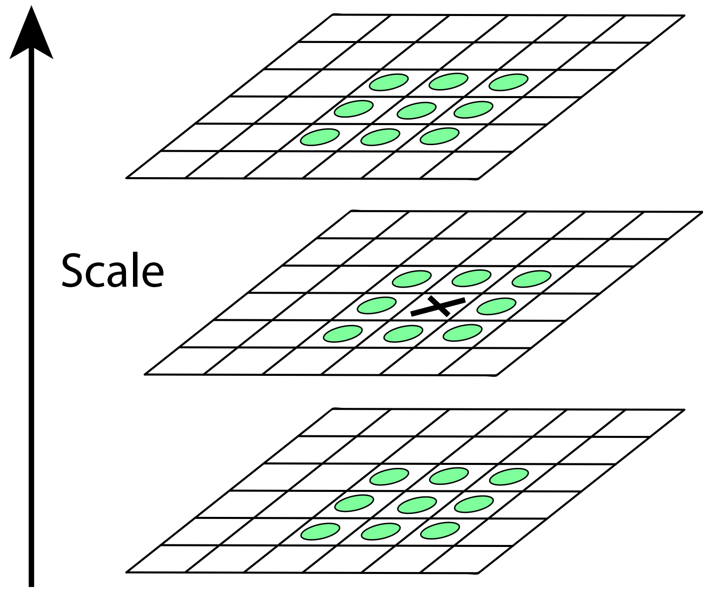
Are eigenvector stable under scalings? Yes
Are eigenvalues stable under scalings? No

A solution to scale

search over image pyramid scales

$$A(x, y, \sigma) = \sum_{x,y} \begin{bmatrix} \mathbf{I}_x(\sigma)^2 & \mathbf{I}_x\mathbf{I}_y(\sigma) \\ \mathbf{I}_y\mathbf{I}_x(\sigma) & \mathbf{I}_y^2(\sigma) \end{bmatrix}$$

A solution to scale



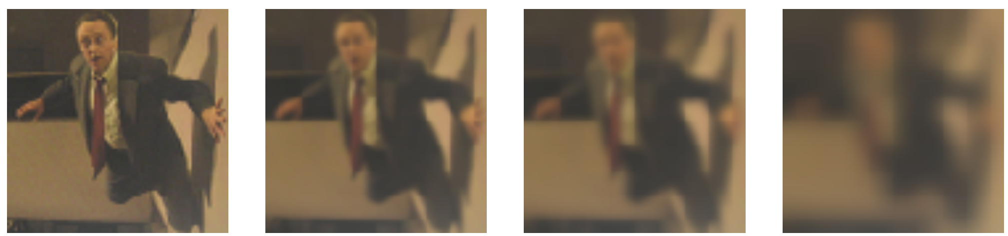
$$\text{cornerness}(x, y, \sigma) = det(A(x, y, \sigma)) - \alpha Trace^2(A(x, y, \sigma))$$

Look for local maxima in (x,y,sigma)

Annoying “details”

1. Positions across scales don’t align

Soln: construct blurred versions of image



2. Gradients across scales aren’t comparable
(gradients always smaller on blurred images)

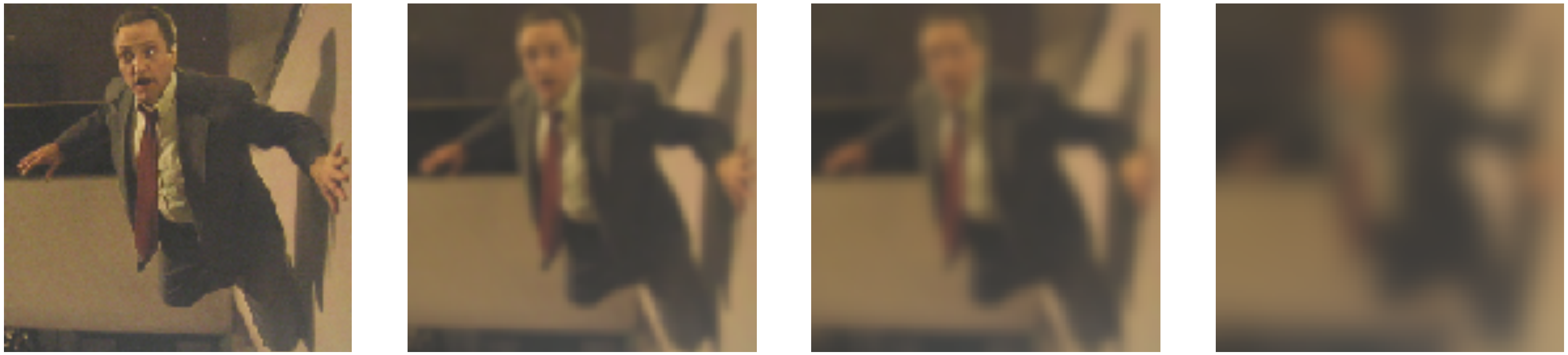
Soln: multiply gradients by scale factor

Scale-space theory: A basic tool for analysing structures at different scales

Tony Lindeberg

Computational Vision and Active Perception Laboratory (CVAP)
Department of Numerical Analysis and Computing Science

Putting it all together: Harris-Laplacian detector



$$A(x, y, \sigma_I, \sigma_d) = \sigma_D^2 G(\sigma_I) * \begin{bmatrix} \mathbf{I}_x(\sigma_D)^2 & \mathbf{I}_x\mathbf{I}_y(\sigma_D) \\ \mathbf{I}_y\mathbf{I}_x(\sigma_D) & \mathbf{I}_y^2(\sigma_D) \end{bmatrix}$$

Relate Gaussian for *integration* with Gaussian for computing *derivatives*

Heuristic: $\sigma_D = .7\sigma_I$

https://en.wikipedia.org/wiki/Harris_affine_region_detector

“Sub-pixel” accuracy across sigma (and x,y)

1. Optimize cornerness(x,y,sigma) over discrete set of locations and scales

2. Fine-tune “sub-pixel” accuracy by iterating the following:

   i. Given (x,y), we can find maximal sigma with finer search

   ii. Given sigma, find maximal (x,y) of cornerness

Repeat (i,ii) over local neighborhoods of (sigma,x,y) until convergence

Scale selection in 2D

Lindeberg et al., 1996

$f(I_{i_1 \ldots i_m}(x,\sigma))$

Slide from Tinne Tuytelaars

Scale selection in 2D

Function responses for increasing scale

Scale trace (signature)

$f(I_{i_1 \dots i_m}(x,\sigma))$

Scale selection in 2D

Function responses for increasing scale

Scale trace (signature)

$f(I_{i_1 \ldots i_m}(x, \sigma))$

Scale selection in 2D

Function responses for increasing scale

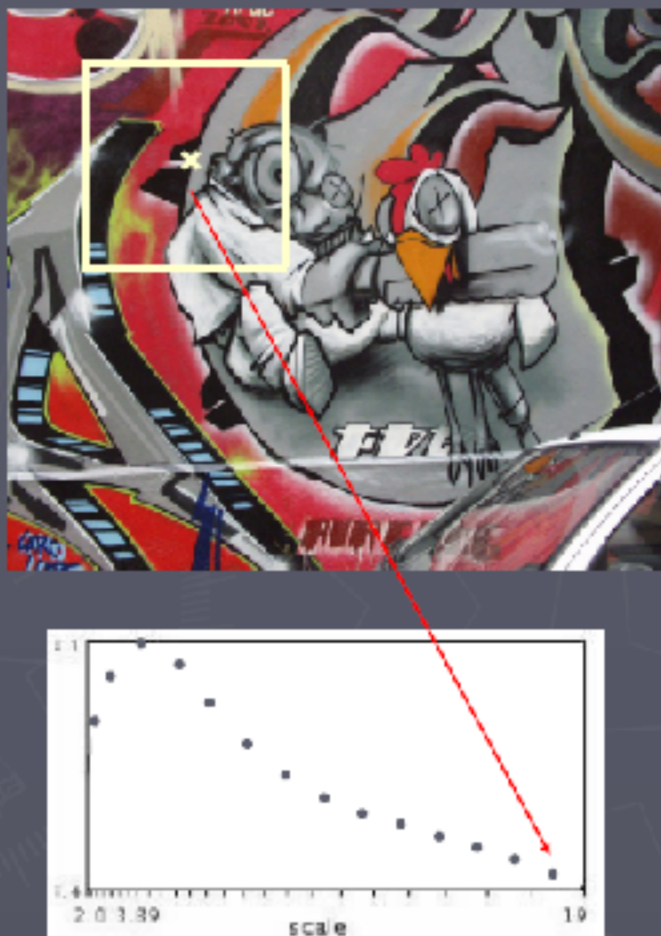
Scale trace (signature)

$f(I_{i_1 \ldots i_m}(x, \sigma))$

Scale selection in 2D

Function responses for increasing scale

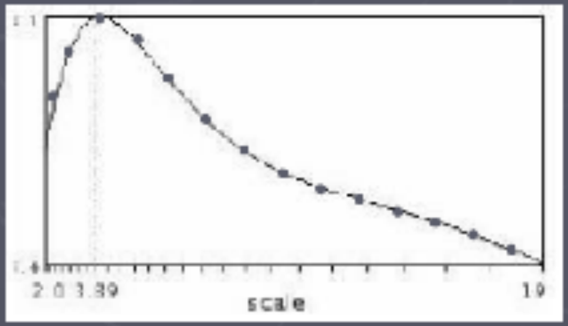
Scale trace (signature)



$f(I_{i_1 \ldots i_m}(x,\sigma))$

Scale selection in 2D

Function responses for increasing scale
Scale trace (signature)

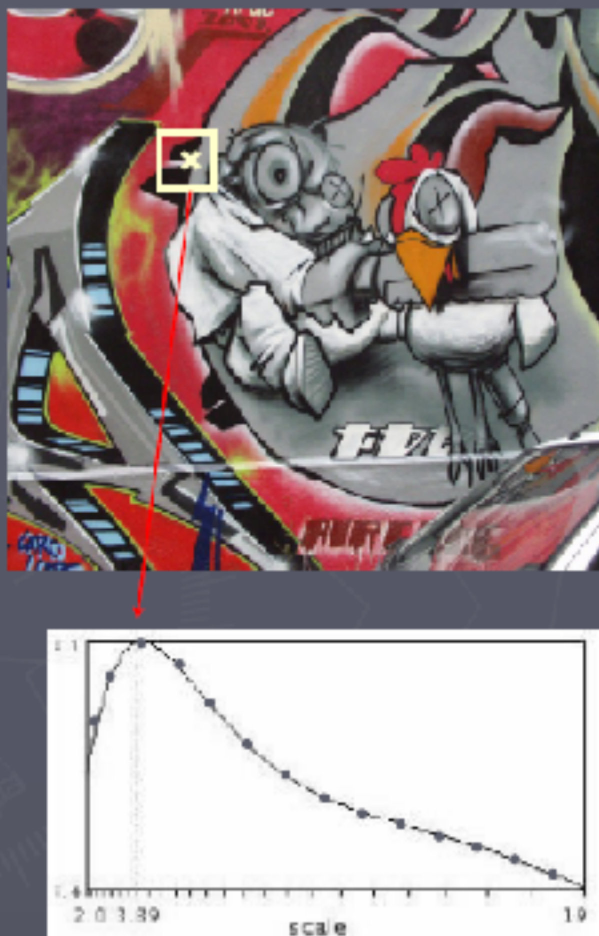


$f(I_{i_1 \ldots i_m}(x,\sigma))$

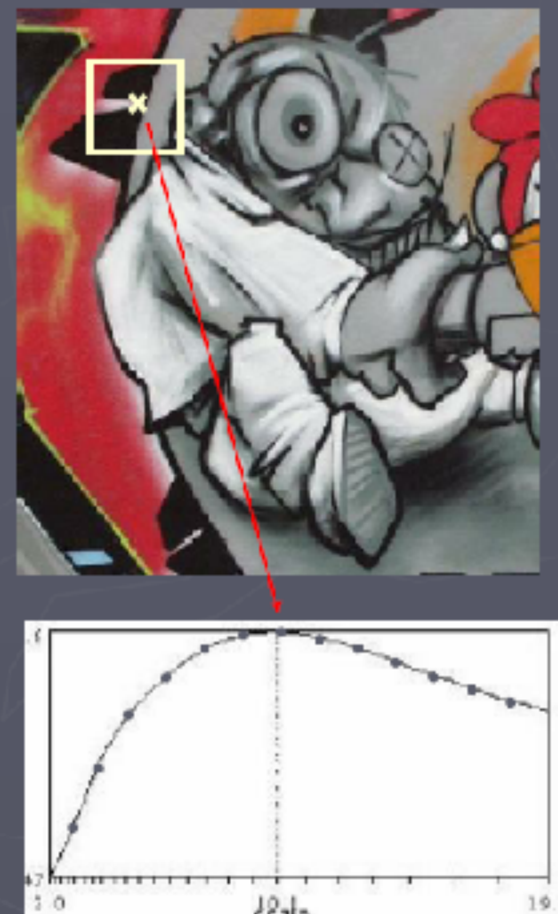
Scale selection in 2D

Function responses for increasing scale

Scale trace (signature)



$f(I_{i_1 \ldots i_m}(x,\sigma))$



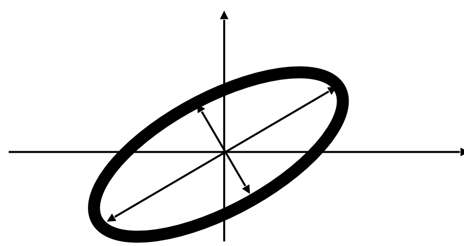
$f(I_{i_1 \ldots i_m}(x',\sigma'))$

Extension 1: anisotropic scale

Need richer description of "neighborhood" or scale

Replace scalar σ with $\sum$

(e.g., scale differently long x and y, or even a diagonal axis)



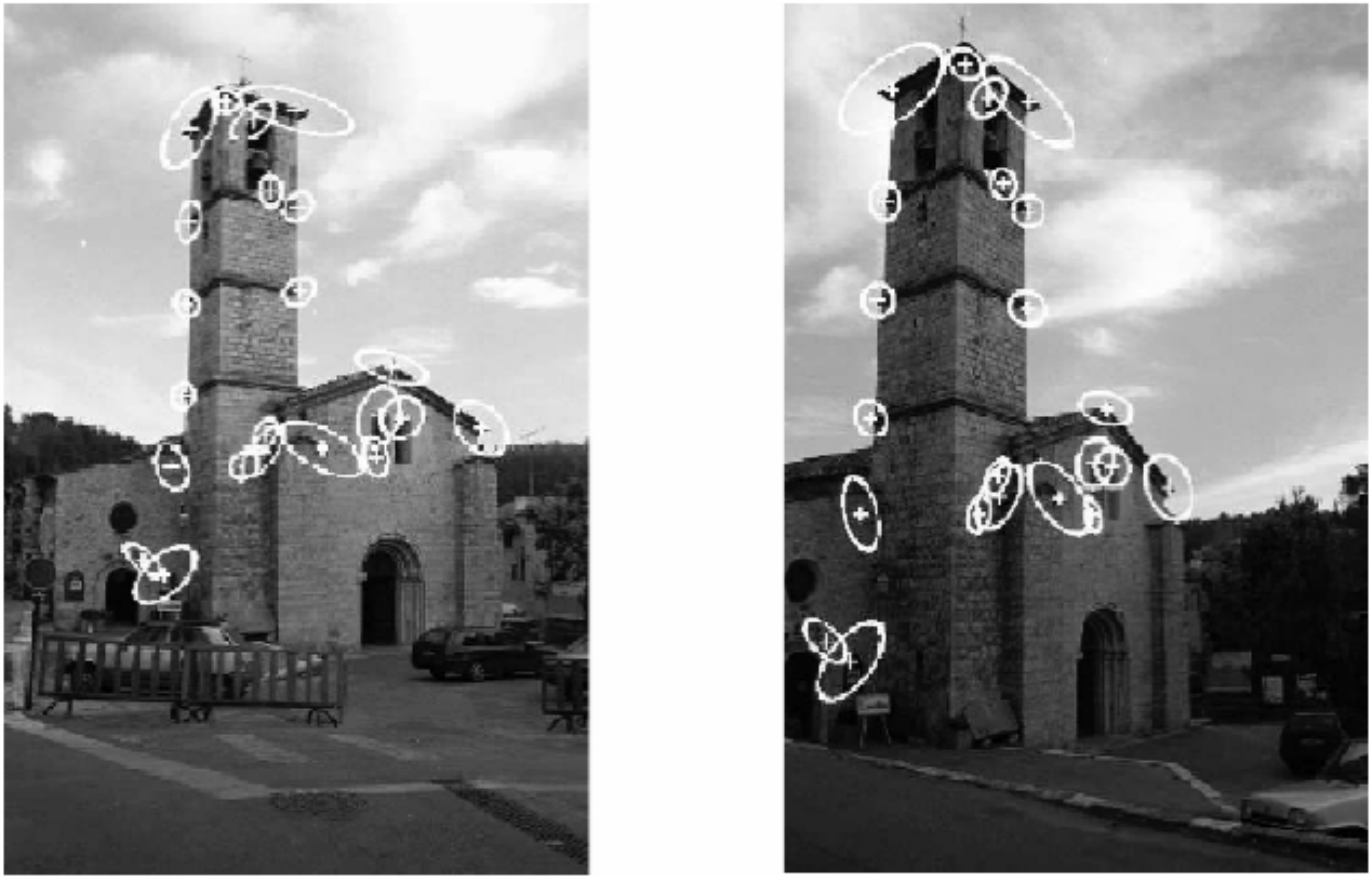
1. Optimize cornerness(x,y,sigma) over discrete set of locations and scales

2. Fine-tune "sub-pixel" accuracy by iterating the following:

   i. Given (x,y), find maximal Sigma with local search
   ii. Given Sigma, find maximal (x,y) of cornerness

Affine Invariance

Application: Finding correspondences

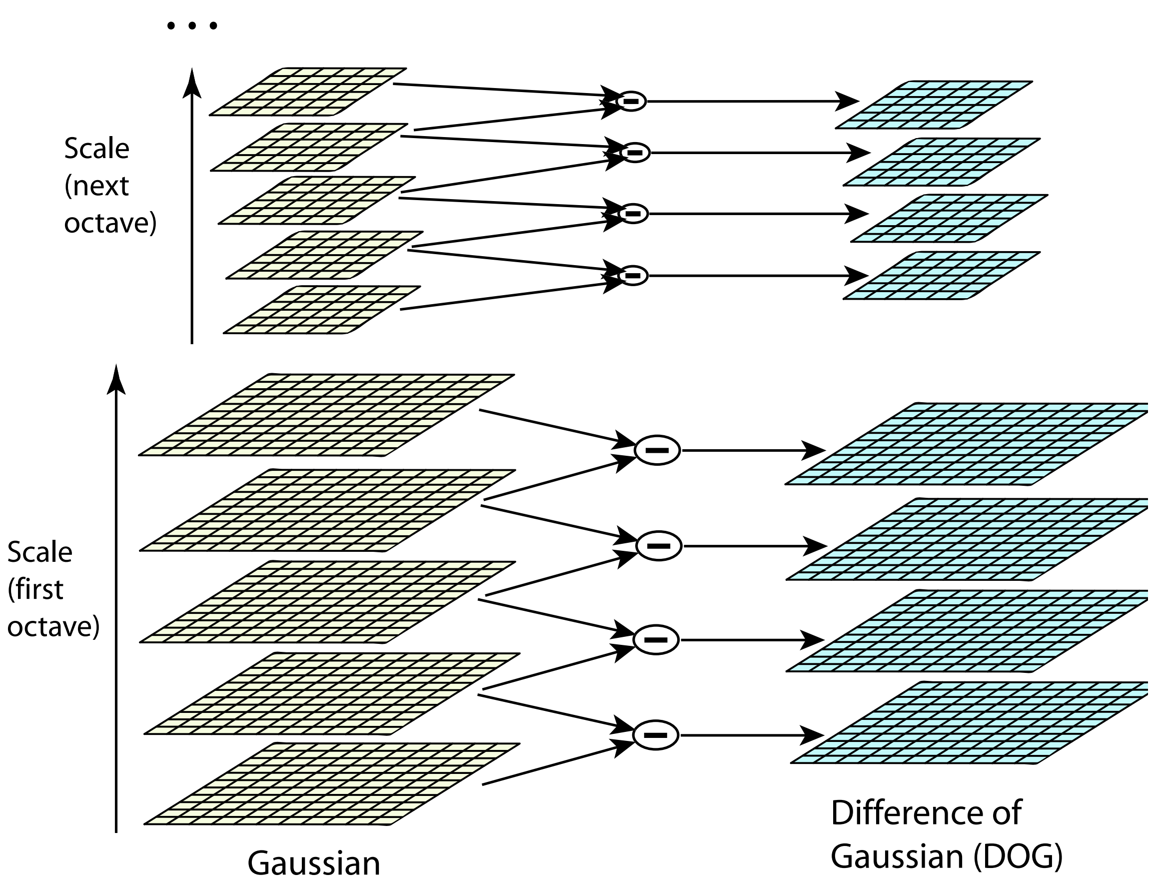


Final matches: 32 correct correspondences
Scale: 4.9
Rotation: 19^o

Example from *Mikolajczyk and Schmid 2004*

Extension 2: directly work with scale-space features or "blobs"

https://en.wikipedia.org/wiki/Scale-invariant_feature_transform



$$D(x, y, \sigma) = (G(x, y, k\sigma) - G(x, y, \sigma)) * I(x, y)$$

$k = 2^{\frac{1}{s}}$ where s = # levels in an octave

Look for “blob detections” that are

locally maximal, high confidence, and localizeable

Local maxima of D(x,y,sigma)

D(x,y,sigma) > thresh

$$\begin{bmatrix} D_{xx} & D_{xy} \\ D_{yx} & D_{yy} \end{bmatrix}$$

min eigenvalue of Hessian > thresh

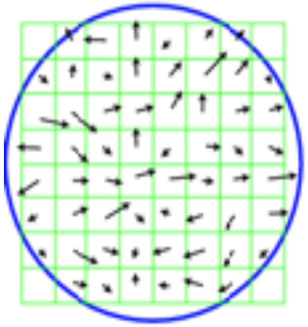
Added benefit of Hessian: use second-order taylor expansion to get “subpixel” accuracy

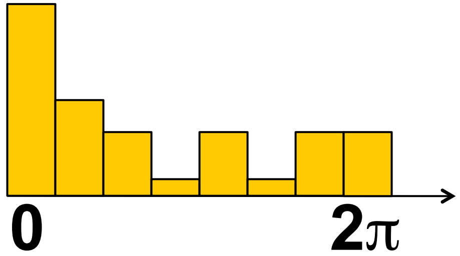
https://en.wikipedia.org/wiki/Scale-invariant_feature_transform

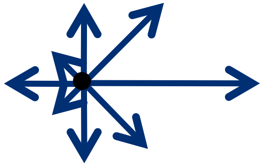
Alternative approach for rotation invariance

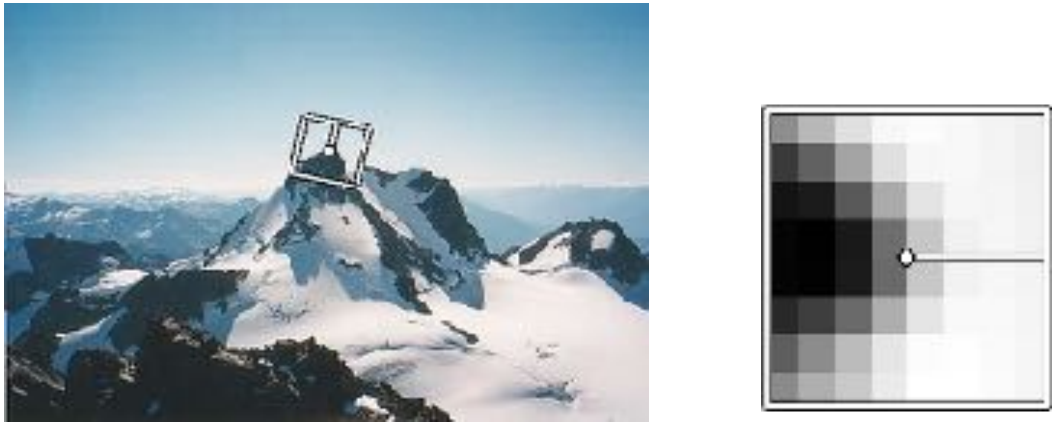
(Lowe, SIFT)

Compute gradients for all pixels in patch. Histrogram (bin) gradients by orientation







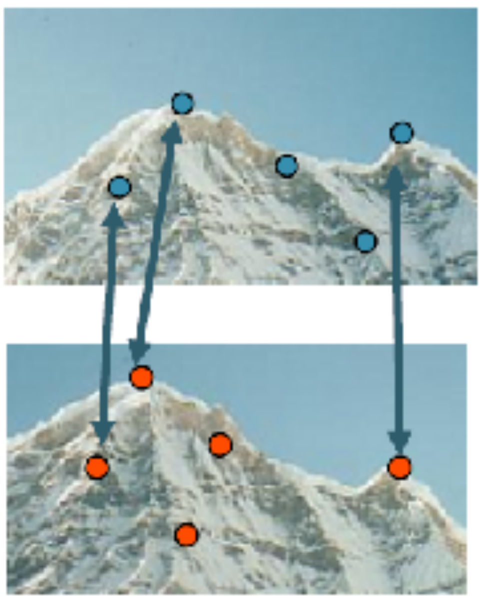


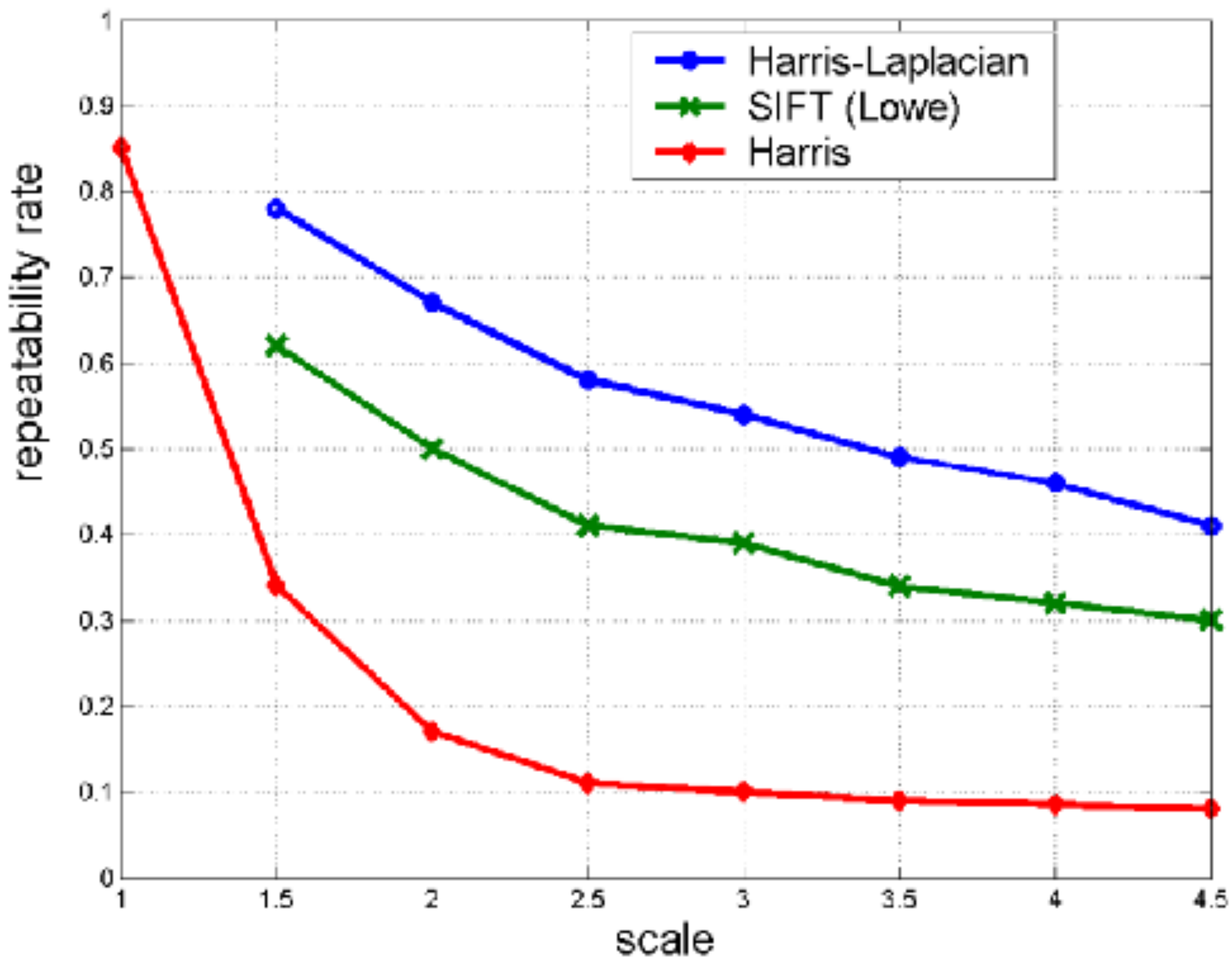
(I prefer this because you can look for multiple peaks)

Comparison

$$\frac{\#\ \text{correspondences}}{\#\ \text{possible correspondences}}$$

(points present)





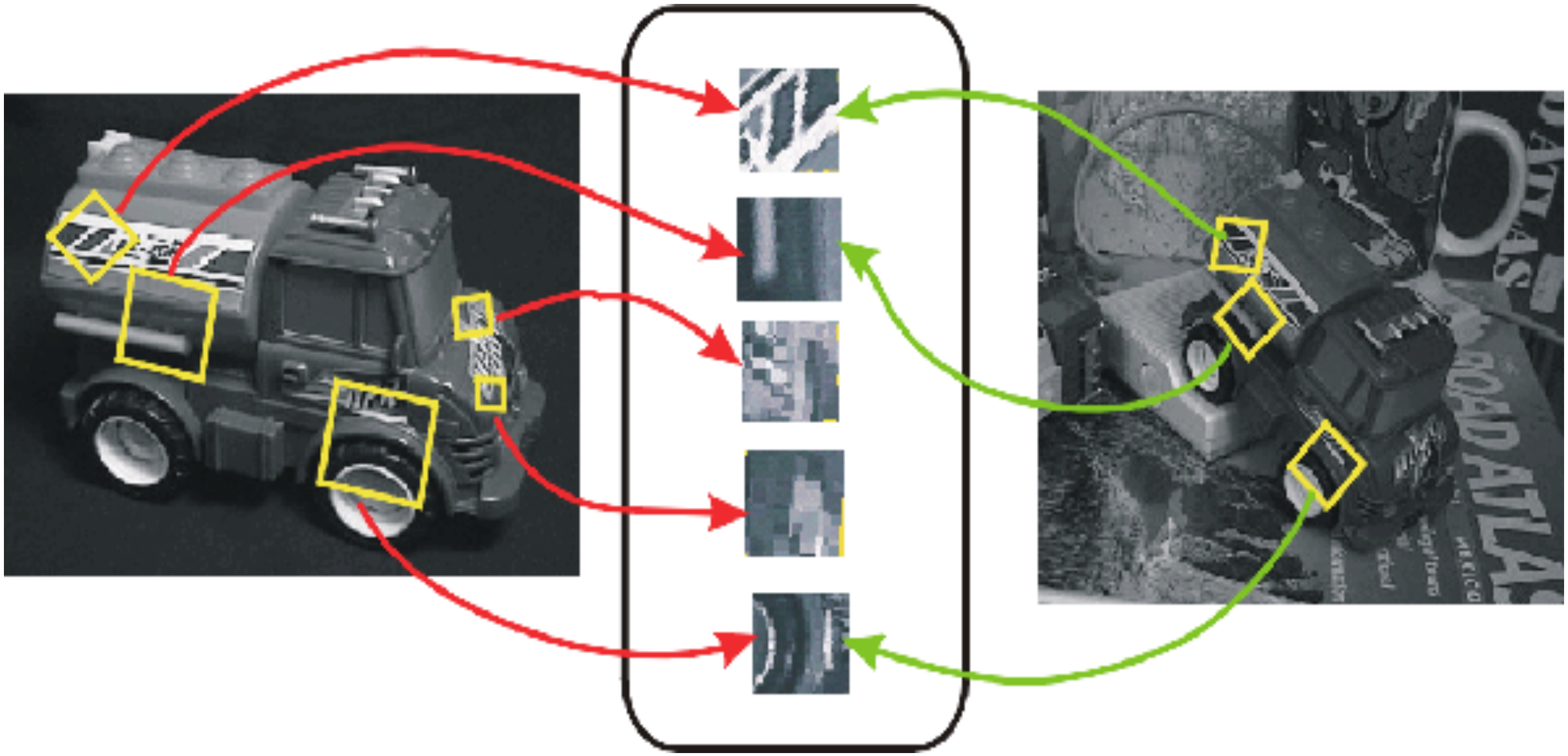
References

- K. Mikolajczyk and C. Schmid. A performance evaluation of local descriptors. Pattern Analysis and Machine Intelligence (PAMI), 27(10):31–47,2005.
- David G. Lowe. Distinctive Image Features from Scale-Invariant Keypoints. IJCV (International Journal of Computer Vision), 2004
- K.Mikolajczyk and C.Schmid. Scale & Affine Invariant Interest Point Detectors. IJCV, Vol. 60, No. 1, 2004.

Software can be downloaded from Schmid’s and Lowe’s pages

- T. Lindeberg. Feature detection with automatic scale selection'. International Journal of Computer Vision, vol 30, number 2, pp. 77--116, 1998.
- T. Lindeberg. Scale-Space Theory in Computer Vision, Kluwer Academic Publishers, Dordrecht, Netherlands, 1994.

Coordinate frames



Represent each patch in a canonical scale and orientation (or general *affine* coordinate frame)

$$d(p_1, p_2) = \left\| \begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix} - \begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix} \right\|$$

Scale Invariant Feature Transform

Basic idea:

- Take 16x16 square window around detected feature
- Compute edge orientation (angle of the gradient - 90°) for each pixel
- Throw out weak edges (threshold gradient magnitude)
- Create histogram of surviving edge orientations

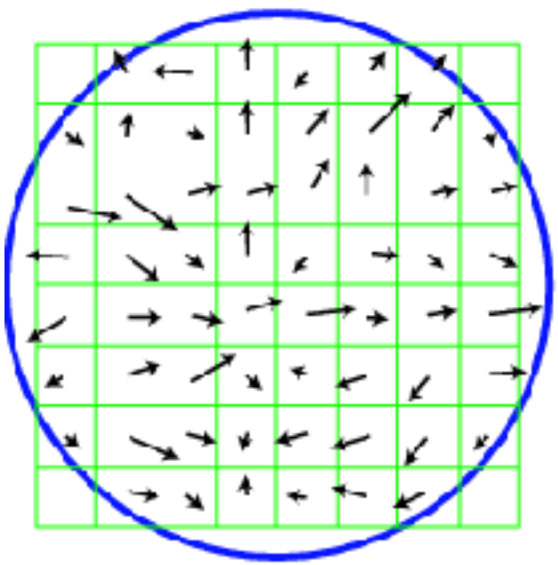
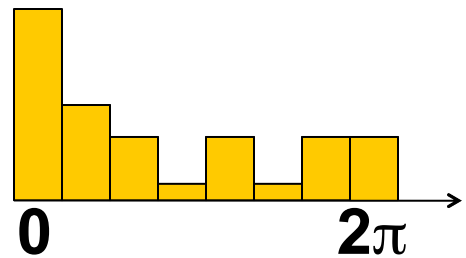
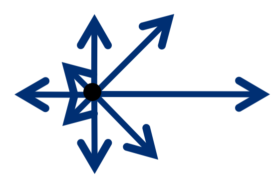


Image gradients



angle histogram

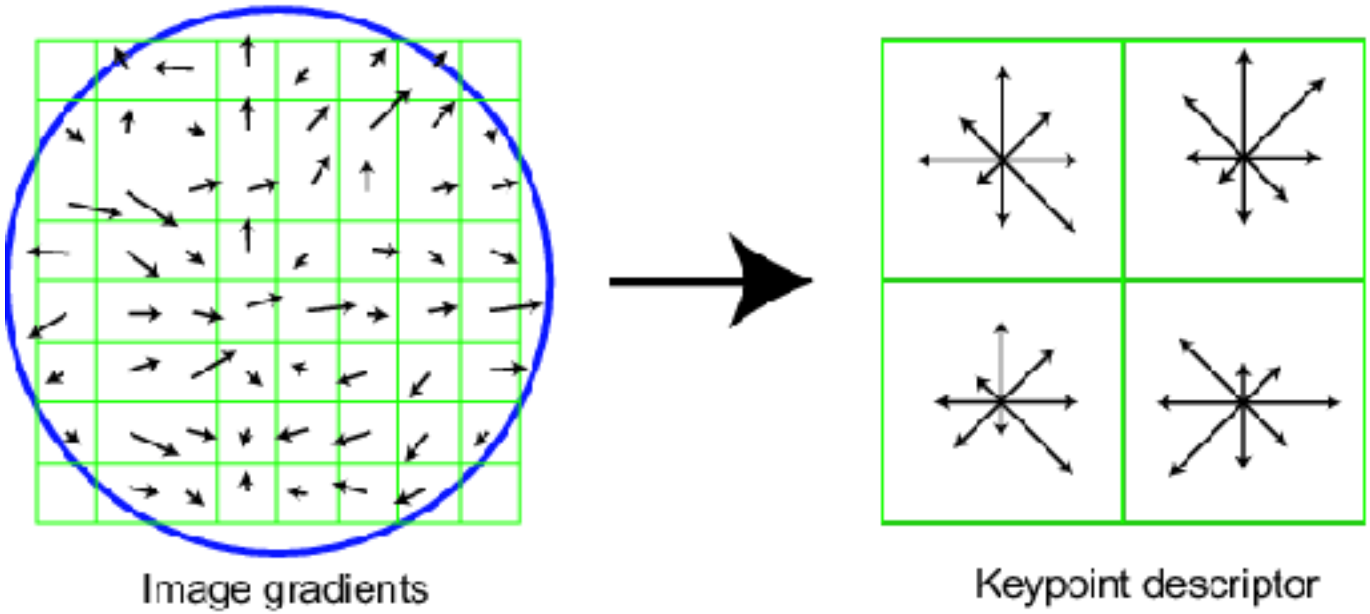


Adapted from slide by David Lowe

SIFT descriptor

Full version

- Divide the 16x16 window into a 4x4 grid of cells (2x2 case shown below)
- Compute an orientation histogram for each cell
- 16 cells * 8 orientations = 128 dimensional descriptor



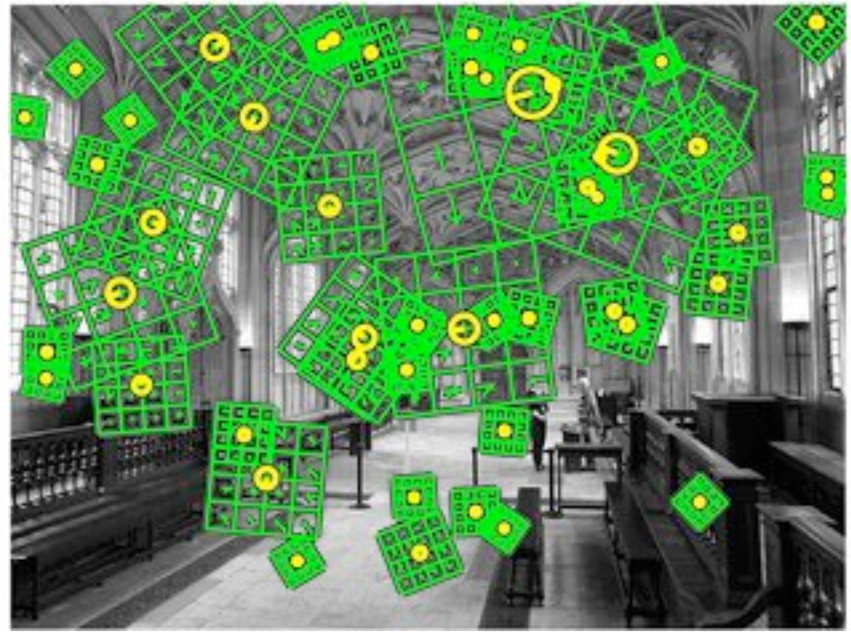
Adapted from slide by David Lowe

Properties of SIFT

Extraordinarily robust matching technique

- Can handle changes in viewpoint
  - Up to about 60 degree out of plane rotation
- Can handle significant changes in illumination
  - Sometimes even day vs. night (below)
- Fast and efficient—can run in real time
- Lots of code available
  - http://people.csail.mit.edu/albert/ladypack/wiki/index.php/Known_implementations_of_SIFT


http://www.vlfeat.org/overview/sift.html



We'll discuss many more on Thursday!